Dynamics for Mechatronics Engineers,

Concepts and Examples- Part II

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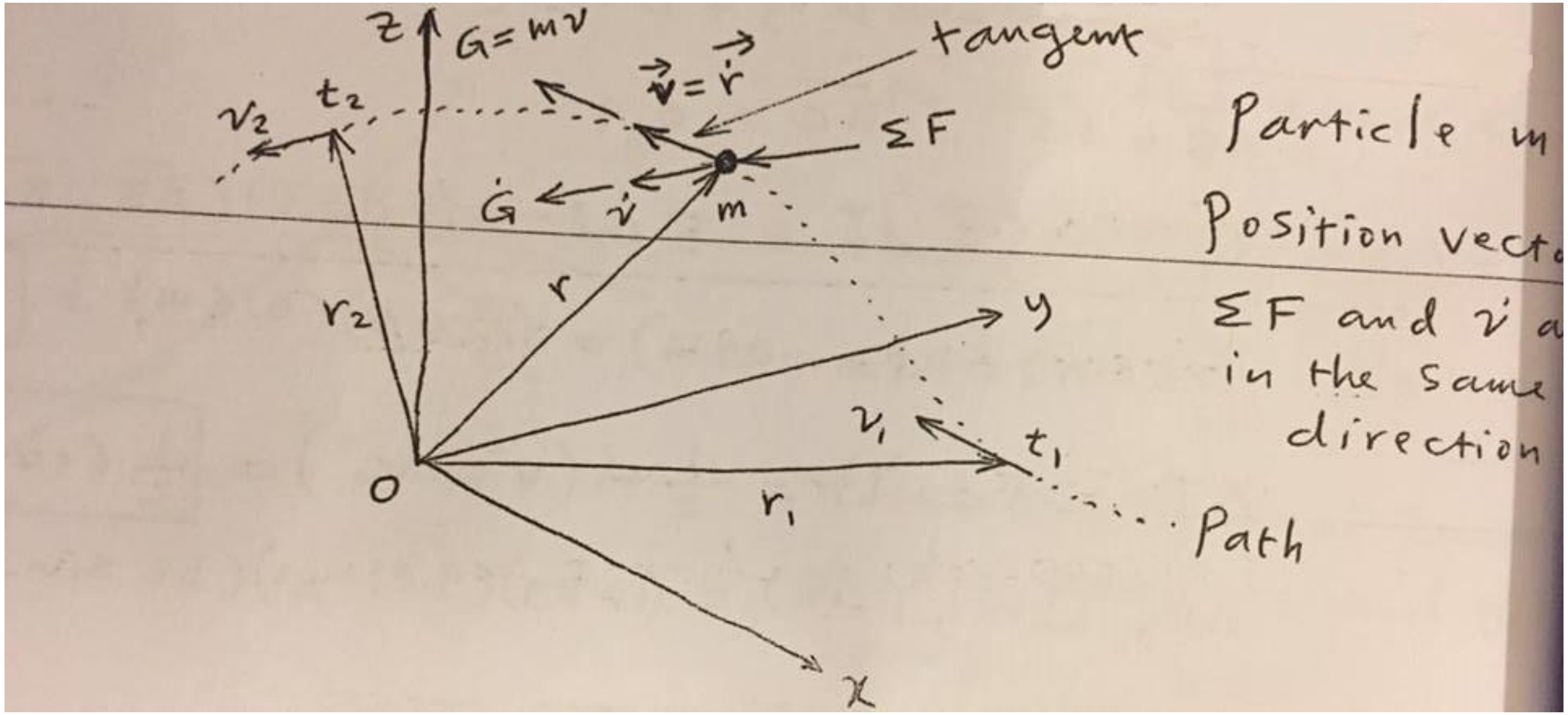
Impulse And Momentum

The equs of impulse and momentum are obtained by integrating the equ of motion wrt time ,whereas, the work-energy equ were obtained by integrating the equ of motion wrt displacement.

Impulse and momentum equs are used to solve problems in which forces act over specified periods of time

Linear Impulse And Linear Momentum



-The basic equation of motion for the particle is :-

$\Sigma F = mv' = d/dt(mV) = G' \longrightarrow \quad \Sigma F = G'$

-G is the linear momentum , G=mV
The resultant of all forces acting on a particle equals its time rate of change of linear momentum unit of G is kg.m/s ≡ N.s

-The resultant force **ΣF and G' directions coincide with the direction of acceleration ΣF , G and (V=a) have the same direction scalar equations of G'**

$\Sigma Fx = G'x$ $\Sigma Fy = G'y$ $\Sigma Fz = G'z$

-To find the effect of ΣF over a fiuire period of time , integrate $\Sigma F = G'$ wrt time .

$\Sigma F = d/dt\,(G)$ → $\Sigma F\,dt = dG$

→ $\int \Sigma F\,dt = \int dG = G1 - G2 = \Delta G$

→ $G2 = G1 + \int \Sigma F\,dt$

$G2 = mV2$ **$G1 = mV1$**

-Linear Impulse is determined as the product of force and time . The total linear impulse on an m equals the corresponding change in linear momentum of m .

-The scalar equations of $\Delta G = \int \Sigma F\, dt$ are :-

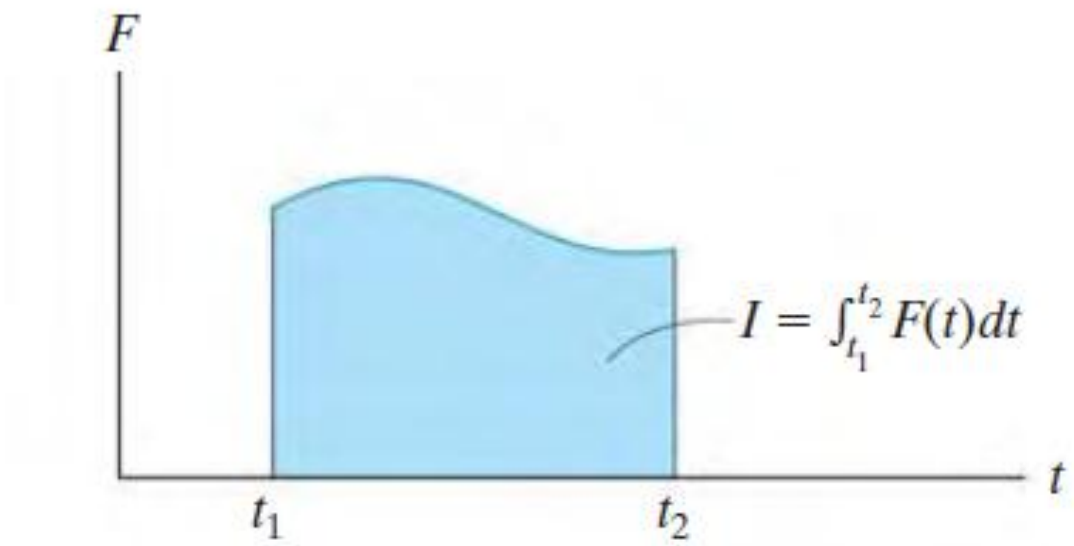
$\int \Sigma Fx\, dt = (mVx)2 - (mVx)1$

$\int \Sigma Fy\, dt = (mVy)2 - (mVy)1$

$\int \Sigma Fz\, dt = (mVz)2 - (mVz)1$

These Impusle – Momentum equations are independent

-Impulse = $\int F\, dt$ = area under the curve



Consevation of linear Momentum (G)

-G is conserved if $\Sigma F = 0$ during the time interval . ⟶ G remains constant

-G can be constant in some directions and changing in other directions .

-For two interacting particles (a and b) , with no external forces :-

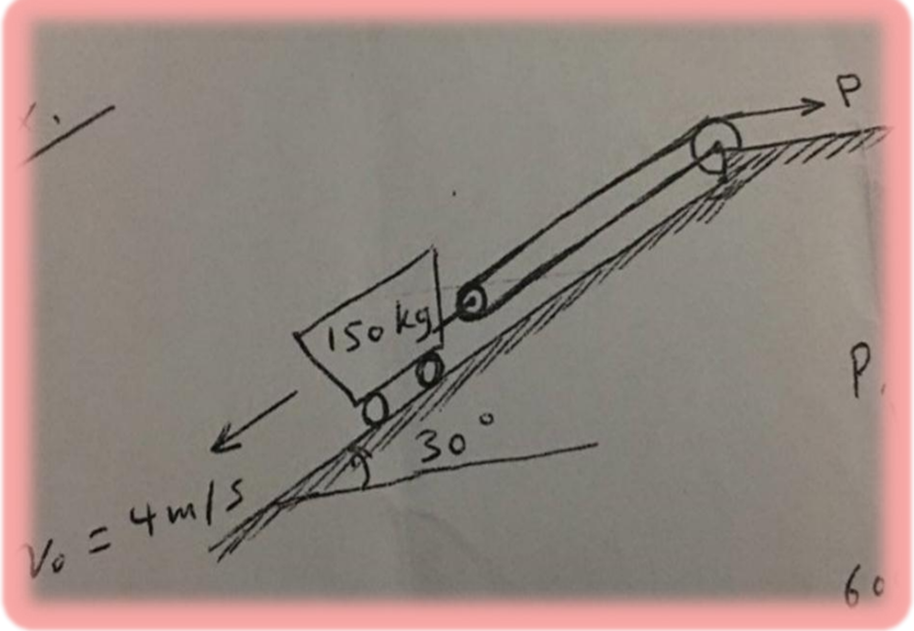
$\Delta(Ga + Gb) = 0$ ⟶ $\Delta G\ tot = 0$ ⟶ $G1 = G2$

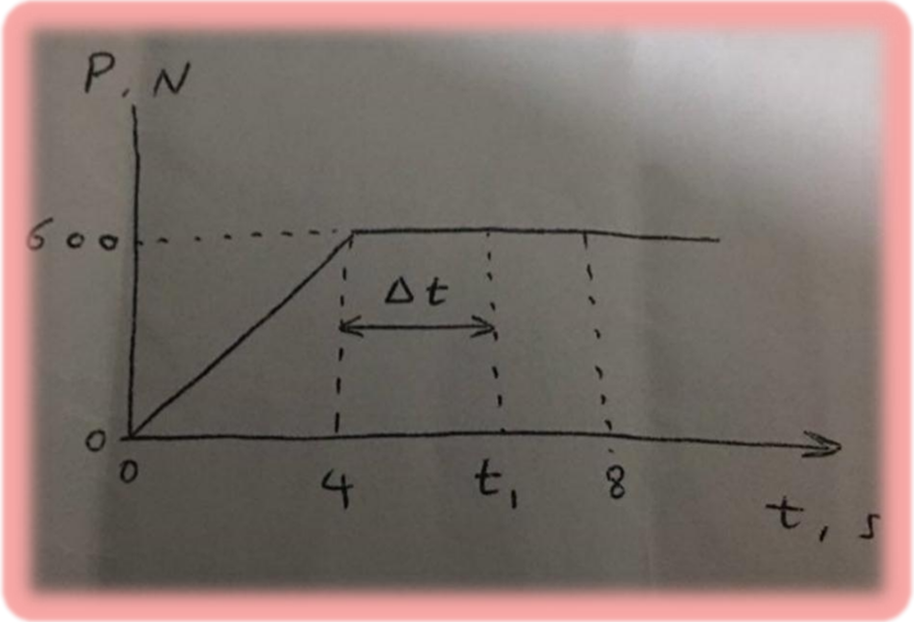
Example :-

Find :

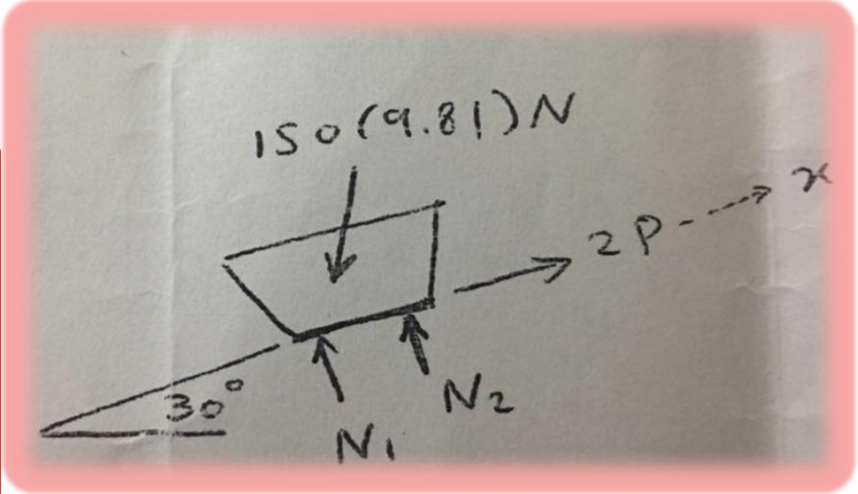
a) t1 when the skip reverse it's direction ?

b) Velocity of the skip at t=8 sec





Solution :-



a) Skip reverse direction when V=0

Assume that V=0 , at t=4 +Δt

Use Impulse-Momentum equation

$\int_0^{t1} \sum \boldsymbol{f}_{\mathbf{x}}$ dt = m $\Delta \mathbf{v}_{\mathbf{x}}$

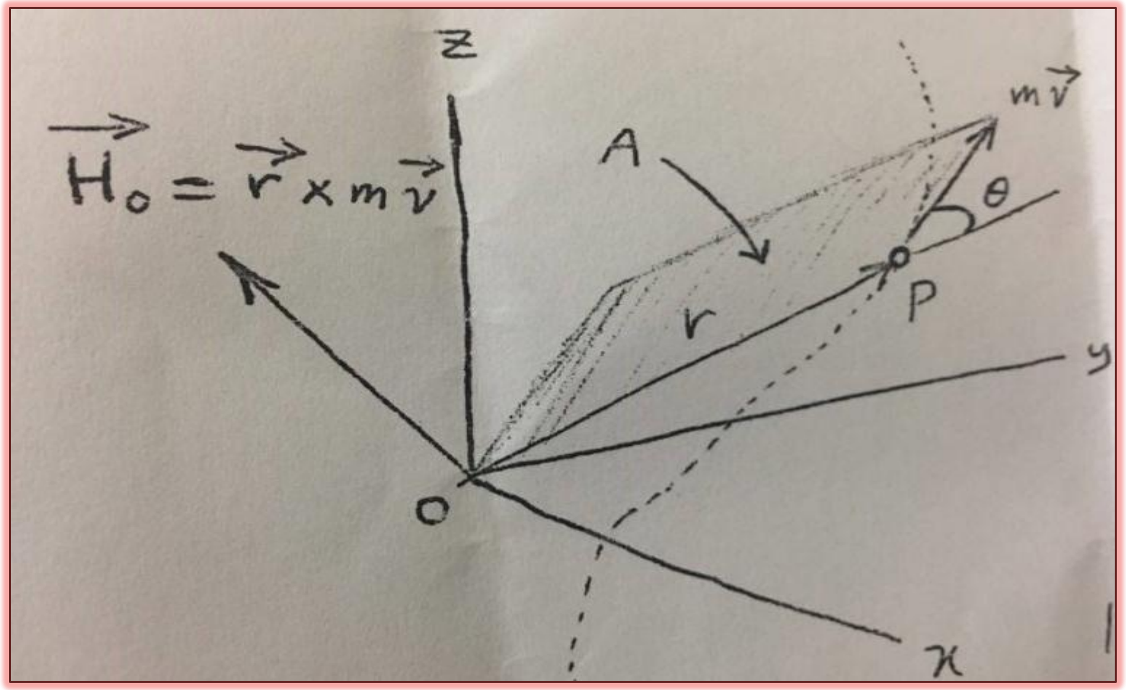
=2*0.5*4*600+2*600*Δt -150*9.81*$\cos 60$ *(4+ Δt)
=150(0-(-4)) => 464 Δt = 1143 => Δt=2.46s

t1=4+2.46 = 6.46s

b) $\int_0^{8} \sum \boldsymbol{f}_{\mathbf{x}}\, \boldsymbol{dt}$ = m Δ $\mathbf{v}_{\mathbf{x}}$ => 2*0.5*4*600+2*(8-4)*600-150*9.81*$\cos 60$*8
=150(v – (-4)) =>
150 v =714 => V=4.76 m/s

$\int_{t1}^{8} \sum \boldsymbol{f}_{\mathbf{x}}\, \boldsymbol{dt}$ = m Δ $\mathbf{v}_{\mathbf{x}}$ => (2*600*(8-6.46)-150*9.81* $\cos 60$ *(8-6.46)) =>
150(v-0)=150v **{same result }**

Angular Impulse and angular momentum

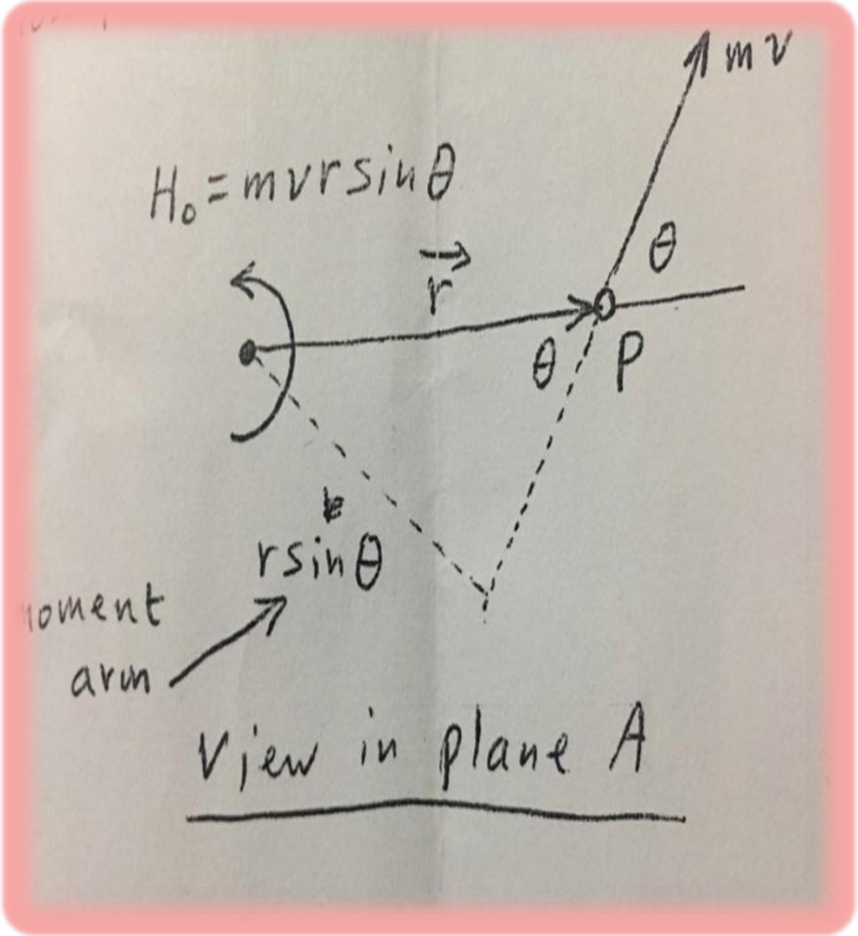


$\vec{v}=\vec{r}'$
$\vec{G}$=m$\vec{v}$
↑
Linear Momentum
The angular momentum H_0 is defined as the moment of the linear momentum m$\vec{v}$ about o

For particle P: $\vec{H}_0 = \vec{r} \text{ X } m\vec{v}$

↑ **Cross product**

$\boldsymbol{H_0}$ is a vector perpendicular to plane A , right hand rule is used to determine the sense of $\boldsymbol{H_0}$



Scaler components of angular momentum :

$$\vec{H}_0 = \vec{r} \times m\vec{v} = m\,(x\hat{i} + y\hat{j} + z\hat{k})*(v_x\, x\hat{i} + v_y\, y\hat{j} + v_z\, z\hat{k})$$

$$= m\,[x\, v_y\, \hat{k} - x\, v_z\, \hat{j} - y\, v_x\, \hat{k} + y\, v_z\, \hat{i} + z\, v_x\, \hat{j} - z v_y\, \hat{i}]$$

$$=> \vec{H}_0 = m \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix}$$

$$H_x = m(v_x\, y - v_y\, z) \qquad H_y = m(v_x\, z - v_z\, x)$$

$$H_z = m(v_y\, x - v_x\, y)$$

Angular Impulse and angular momentum

$H_X = \mathrm{m}v_z y - \mathrm{m}v_y \mathrm{z}$

$= \mathrm{m}(v_z y - v_y \mathrm{z})$

****

$H_y = \mathrm{m}v_x \mathrm{z} - \mathrm{m}$

$= \mathrm{m}(v_x \mathrm{z} - v_z x)$

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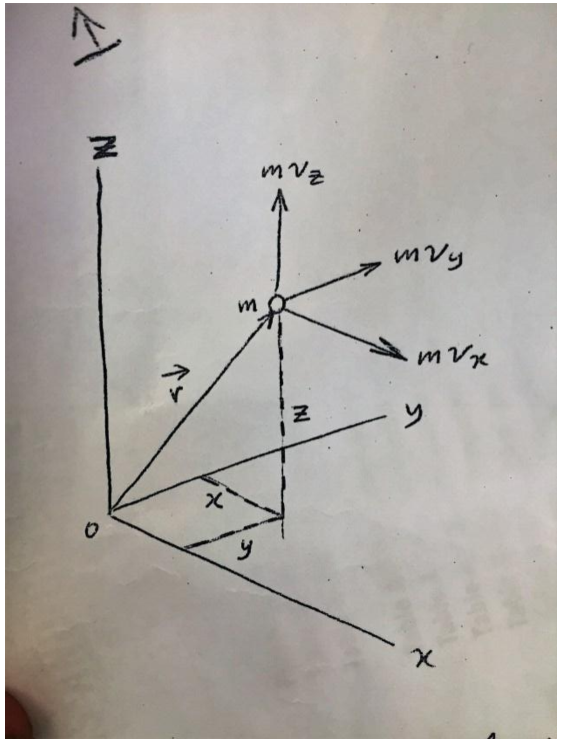
$\mathrm{Hz} = \mathrm{m}v_y x - \mathrm{m}v_x y$

$= \mathrm{m}(v_y x - v_x y)$



H_0 : unit is N.m.s

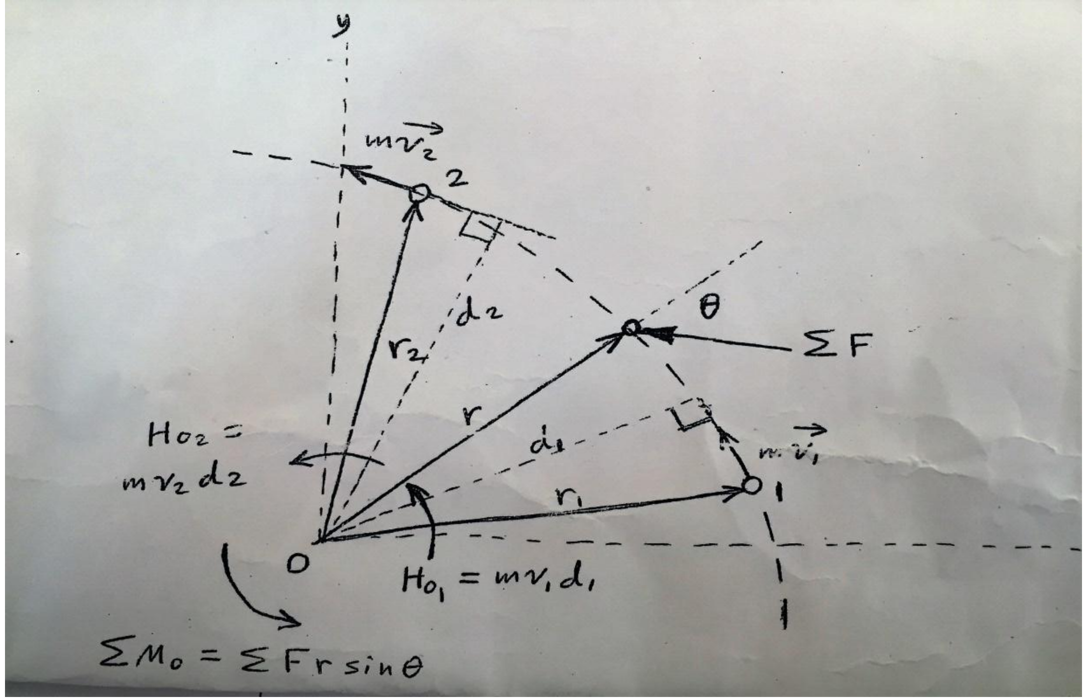
* moment M_0 and angular momentum H_0

****

- $\sum M_0 = \vec{r} \times \sum \vec{f} = \vec{r} \times m\vec{a} = (\vec{r} \times m\vec{v})'$

  $H_0 = \vec{r} \times m_{\vec{v}} :: H_0' = \vec{r}' \times m_{\vec{v}} + \vec{r} \times m_{\vec{v}}'$

  $= \vec{r} \times m_{\vec{v}}'$



Acceleration

- The moment of all forces equals the rate of change of angular momentum:

$$\sum \vec{M}_o = \dot{H}_o$$

- Scalar components:

$$\sum M_{ox} = \dot{H}_{ox} \quad \sum M_{oy} = \dot{H}_{oy} \quad \sum M_{oz} = \dot{H}_{oz}$$

- For a period of time integrate the equation:

$$\sum M_o = \dot{\vec{H}}_o = \frac{d\vec{H}_o}{dt} \rightarrow \sum \vec{M}_o dt = d\vec{H}_o \rightarrow \int_{t_1}^{t_2} \sum \vec{M}_o dt = \int_{H_{o1}}^{H_{o2}} d\vec{H}_o = \vec{H}_{o_2} - \vec{H}_{o_1} = \Delta\vec{H}_o$$

$$\vec{H}_{o_1} = \vec{r}_1 \times m\vec{v}_1 \vec{H}_{o_2} = \vec{r}_2 \times m\vec{v}_2$$

- Angular impulse (N.m.s):

  The total angular impulse equal the change in angular momentum:

$$\vec{H}_{o_2} = \vec{H}_{o_1} + \int_{t_1}^{t_2} \sum \vec{M}_o dt \; \vec{H}_{o_2} - \vec{H}_{o_1} = \int_{t_1}^{t_2} \sum \vec{M}_o dt$$

Constant Acceleration

- **X- component equation:**

$$\int_{t_1}^{t_2} \sum M_{ox} dt = (H_{ox})_2 - (H_{ox})_1$$

$$m(v_{zy} - v_{yz})_2 - m\,(v_{zy} - v_{yz})_1$$

$$\int_{t_1}^{t_2} \sum M_{oy} dt = (H_{oy})_2 - (H_{oy})_1$$

$$\int_{t_1}^{t_2} \sum M_{oz} dt = (H_{oz})_2 - (H_{oz})_1$$

For the figure in page (51)

$\int_{t1}^{t2} \sum M.dt = H_{.2} - H_{.1} \Longrightarrow \int_{t1}^{t2} \sum F * r * sin\Theta\, dt = mv_2 d_2 - mv_1 d_1$

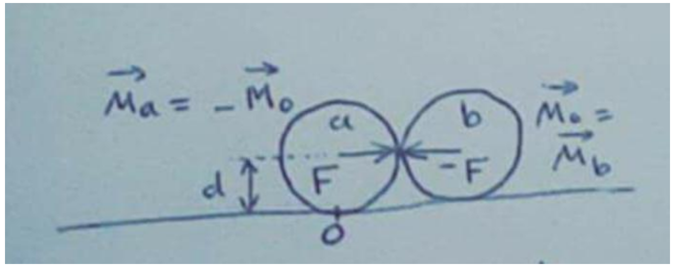
Conservation of Angular Momentum :

If the resultant moment $\sum \vec{M}.=0$ during $\Delta t \Longrightarrow \sum \vec{M}. = \vec{H}. = 0 \Longrightarrow \vec{H}. =$ Constant

Angular momentum is conserved.

$\vec{H}.$ Could be conserved about one axis. But not about another axis.

For two particles a & b, with interactive forces $\vec{F}$ and $-\vec{F}$ between them ;



Moment of unbalanced forces

$\sum \vec{M}.=0$

$\vec{M}_a = -\vec{M}_b$

The principle of conservation of angular momentum :

$$\Delta\vec{H}_a = \vec{H}_{a2} - \vec{H}_{a1} = \int_{t1}^{t2} \sum \vec{M}a\, dt$$

$$\Delta\vec{H}_b = \vec{H}_{b2} - \vec{H}_{b1} = \int_{t1}^{t2} \sum \vec{M}b\, dt$$

add the two equations to get

$$\Delta\vec{H}a + \Delta\vec{H}b = \int_{t1}^{t2} \sum \vec{M}a\, dt + \int_{t1}^{t2} \sum \vec{M}b\, dt = 0$$

$(\vec{M}a = -\vec{M}b)$

$$\Rightarrow \Delta\vec{H}_a + \Delta\vec{H}_b = 0$$

$$\Rightarrow \Delta\vec{H}_{.total} = 0 \Rightarrow \vec{H}_{.1} = \vec{H}_{.2}$$

Kinetics of Particles

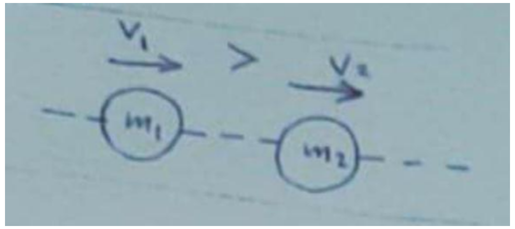
Special Applications

Impact

It refers to the collision between two bodies .

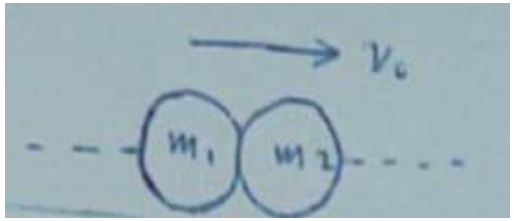
a) Direct central impact :

Before Impact

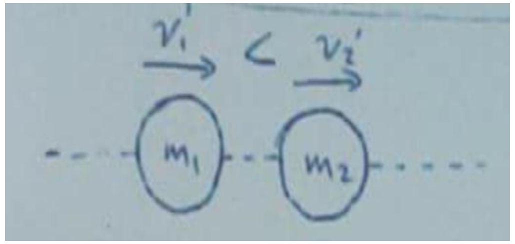


Maximum deformation

During impact



After Impact



Apply the law of conservation of linear momentum :

$$m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2$$

Define the coefficient of restitution e as :

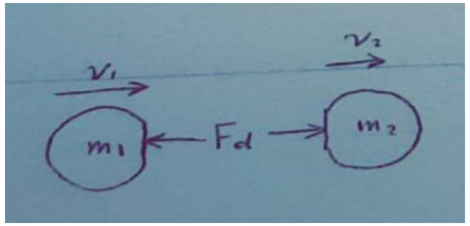
$$e = \frac{Magnitude\ of\ the\ restoration\ impulse}{Magnitude\ of\ the\ deformation\ impulse}$$

For Particle 1

$$e^{(1)} = \frac{\int_{t0}^{t} Fr\ dt}{\int_{0}^{t0} Fd\ dt} = \frac{m_1\,[\,-v'_1 - (-v_0)\,]}{m_1\,[\,-v_0 - (-v_1)\,]} = \frac{v_0 - v'_1}{v_1 - v_0}$$

total time of contact

Deformation time



Deformation period

Restoration period

FOR PARTICLE 2 :

$$e = \frac{\int_{t_0}^{t} F_r \, .dt}{\int_{0}^{t_0} F_d \, .dt} = \frac{(m_2)(V'_2 - V_0)}{(m_2)(V_0 - V_2)} = \frac{(V'_2 - V_0)}{(V_0 - V_2)}$$

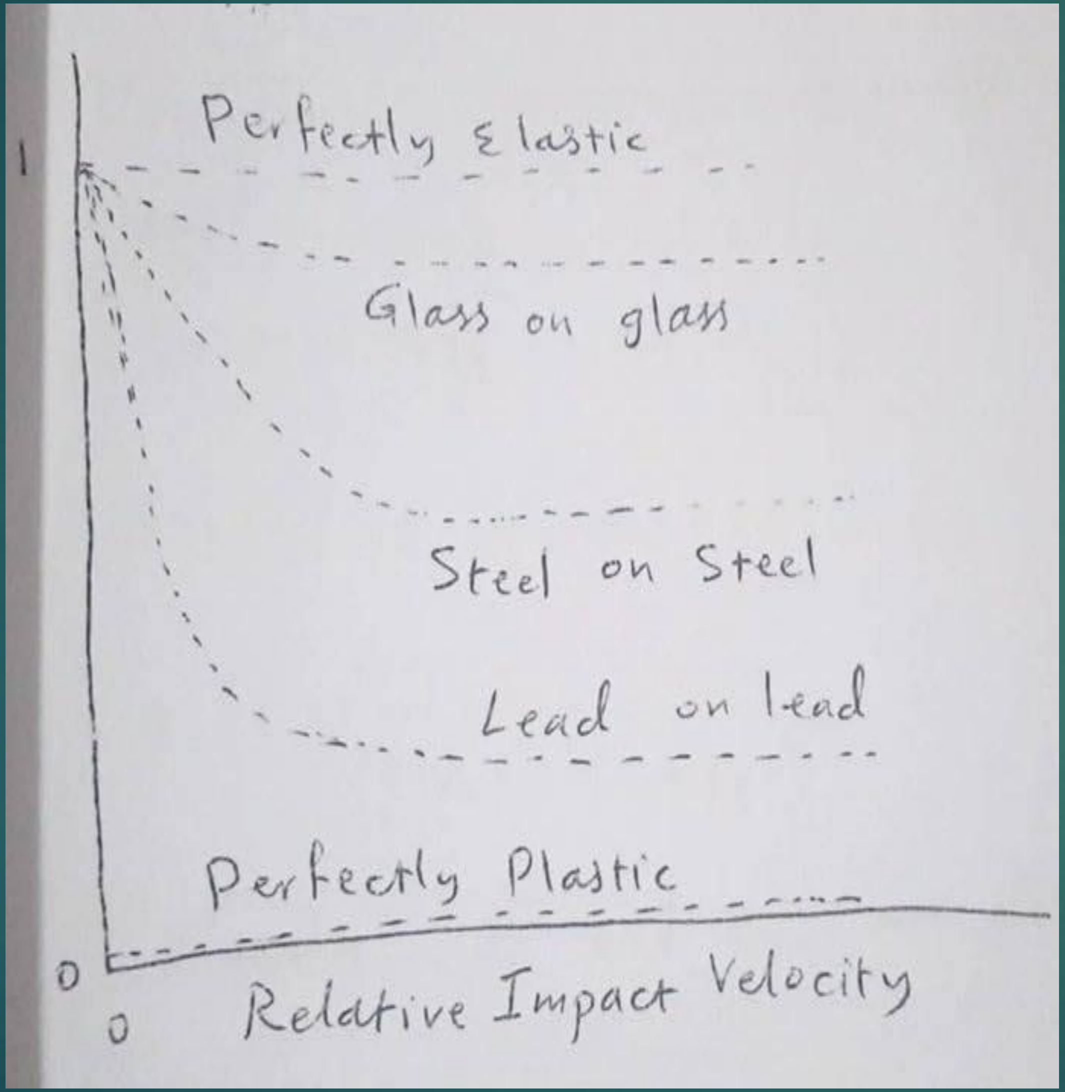
- The change of momentum (and hence ▲V) should be in the same direction as the impulse (and hence the force)

$$e^i = e^{(2)} = \frac{(V_0 - V'_1)}{(V_1 - V_0)} = \frac{(V'_2 - V_0)}{(V_0 - V_2)} \Rightarrow \frac{V_0 - V'_1 + V'_2 - V_0}{V_1 - V_0 + V_0 - V_2} = \frac{V'_2 - V'_1}{V_1 - V_2}$$

$$e = \frac{relative\ velocity\ of\ separation}{relative\ velocity\ of\ approach} = \frac{V'_2 - V'_1}{V_1 - V_2}$$

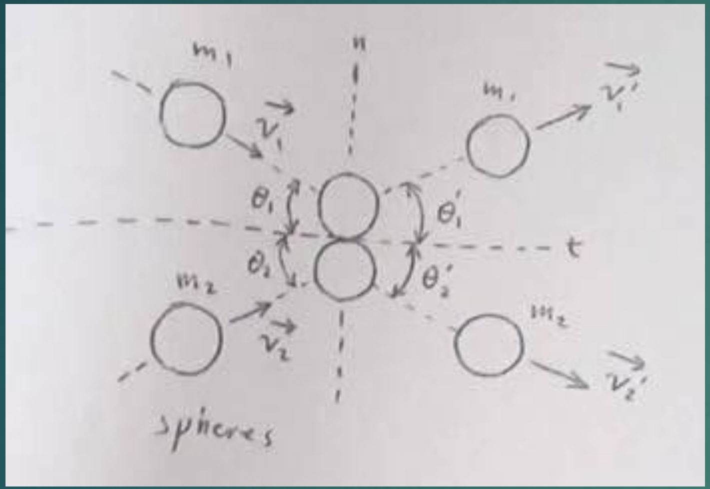
If e = 1 ➡ elastic impact ➡ no energy loss
If e = 0 ➡ inelastic(plastic) impact ➡ max energy loss (particles cling together after impact)

Coefficient of restitution :

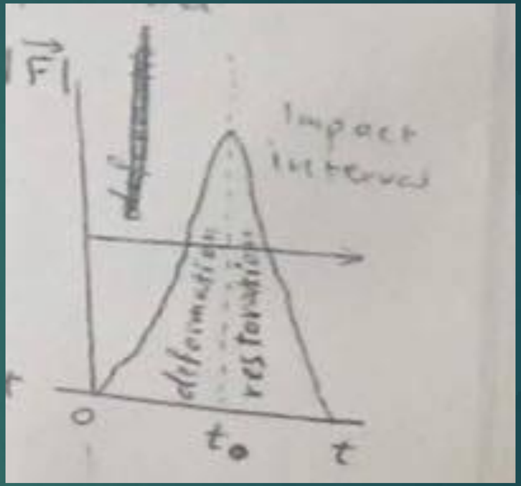


(b) Oblique central Impact :

the initial and final velocities are not parallel



Impact force



$$(v_1)_n = -v_1 \sin\theta_1$$

$$(v_1)_n = v_1 \cos\theta_1$$

$$(v_2)_n = v_2 \sin\theta_2$$

$$(v_2)_t = v_2 \cos\theta_2$$

Given :

$$m_1, m_2, (v_1)_n, (v_1)_t, (v_2)_n, (v_2)_t$$

Unknowns:

$$(v_1')_n, (v_1')_t, (v_2')_n, (v_2')_t$$

Equs:

Conservation of momentum in the n-direction

$$m_1(v_1)_n + m_2(v_2)_n = m_1(v_1')_n + m_2(v_2')_n$$

Conservation of momentum in the (t) direction

$$M_1(v_1) = M_1(v'_1)$$
$$M_2(v_2) = M_2(v'_2)$$

Coefficient of restitution (e)

$$e = \frac{(v'_2)-(v'_1)}{(v_2)-(v_1)}$$

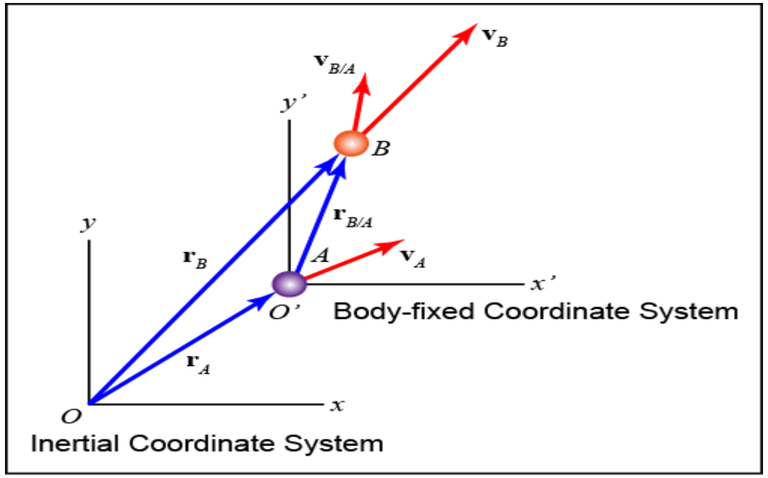
* Note : Finally (θ'_1 & θ'_2) are found using the velocity components.

Relative motion

* It's the consideration of a moving reference system*

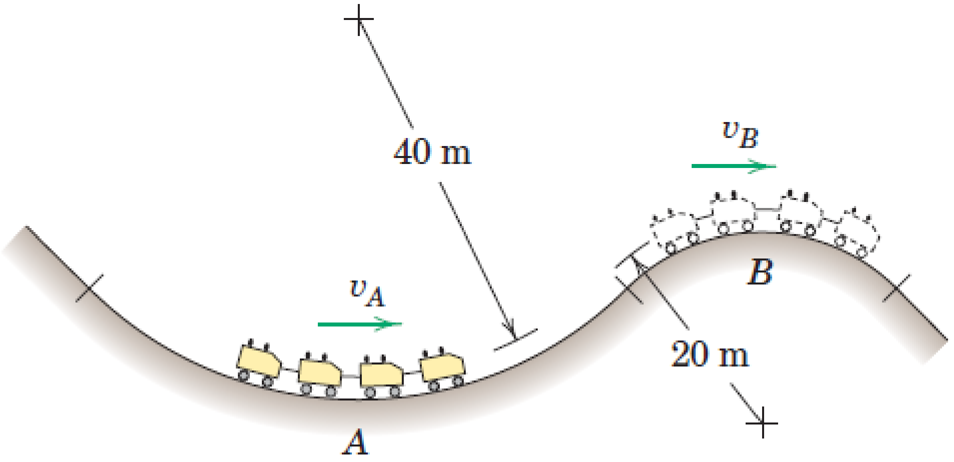
Equations of relative motion:

$$\overrightarrow{a_B} = \overrightarrow{a_A} + \overrightarrow{a_{B/A}}$$
$$\overrightarrow{\sum F} = m * \overrightarrow{a_B}$$
$$\overrightarrow{\sum F} = m * (\overrightarrow{a_A} + \overrightarrow{a_{B/A}})$$



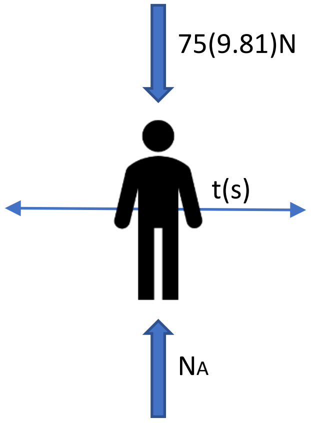
Problem 3/74

The cars of an amusements park have a speed (v_A = 22m/s) at A, and a speed (v_B =12 m/s) at B. If a (75-kg) rider sits on a spring scale (which registers the normal force exerted on it). Determine the scale readings as the car passes points A and B, assume that the person's arms and legs do not support appreciable forces.



Solution :

At point A :

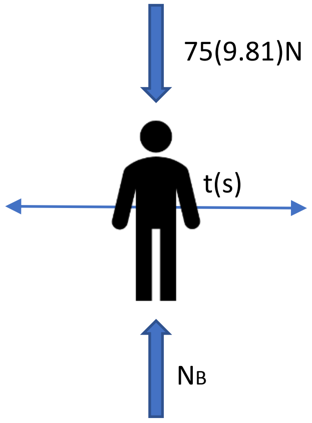


$$\overrightarrow{\sum F} = m * \vec{a}$$

$$N_A - 75(9.81) = 75\frac{22^2}{40}$$

N_A = 1643 (N)

At point B :



$$\overrightarrow{\sum F} = m * \vec{a}$$

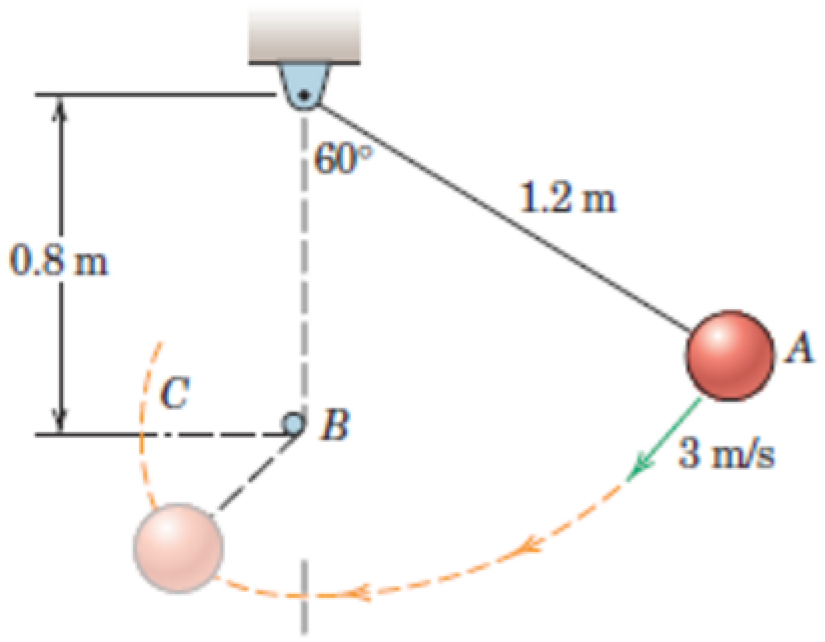
$$-N_B + 75(9.81) = 75\frac{12^2}{20}$$

N_B = 195.8 (N)

Note: static normal force equals the weight (75).(9.81) and it equals 736(N)

Problem 3/129 :

The ball is released from position ***A*** with a velocity of (**3 m/s**) and swings in a vertical plane at the bottom position , the cord strikes the fixed bar at ***B*** , and the ball continues to swing in the dashed arc .
Calculate the velocity ***Vc*** of the ball as it passes position ***C***.



Problem 3/129

Solution :

$U_{1-2} = \Delta T$;

$m\,g\,(0.8 - 1.2\cos(60°)) = 0.5\,m\,(V_c^2 - 3^2)$;

$9.81\,(0.2) = 0.5\,(V_c^2 - 9)$;

$V_c^2 = 12.92$;

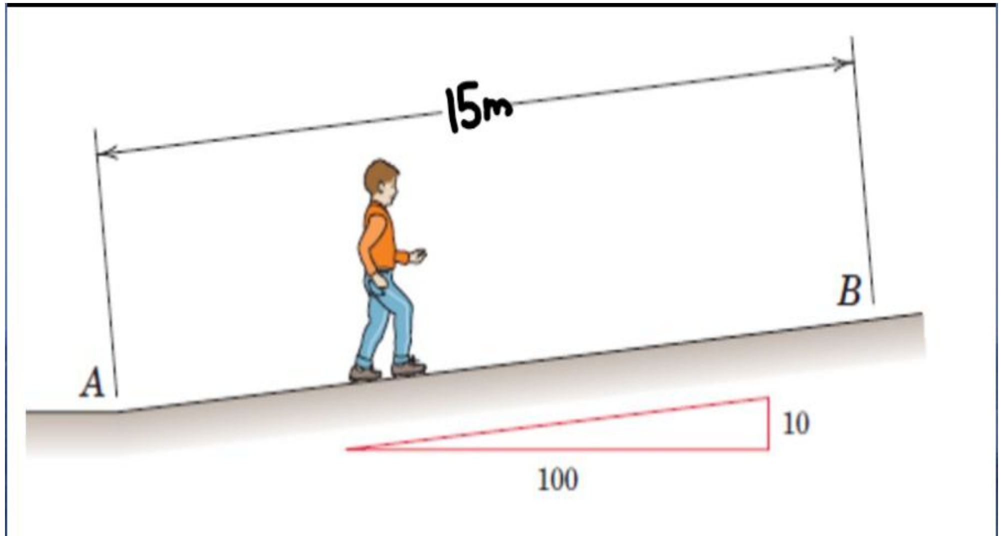
$\mathbf{V_c = 3.59\ m/s}$.

Problem 3/123:

A (***40 – Kg***) boy starts from **rest** at the bottom ***A*** of a **10 – percent** incline and increases his speed at a constant rate to ***8 km/h*** as he passes ***B*** , 15m along the incline from ***A*** .

Determine his power output as he approaches B .

Solution :



Problem 3/123

$V_B = 8/3.6 = 2.22$ m/s

$V_B^2 = V_A^2 + 2a\,\Delta X$

$= 0 + 2a\,\Delta X$

$a = 2.22^2 / 2\,(15) = 0.1646$ m/s²

$\theta = \tan^{-1}(0.1) = 5.71°$

$\overset{+}{\rightarrow} \sum F = ma$

$F - 40\,(9.81)\sin(5.71°) = 40\,(0.1646)$

$F = 45.6$ N

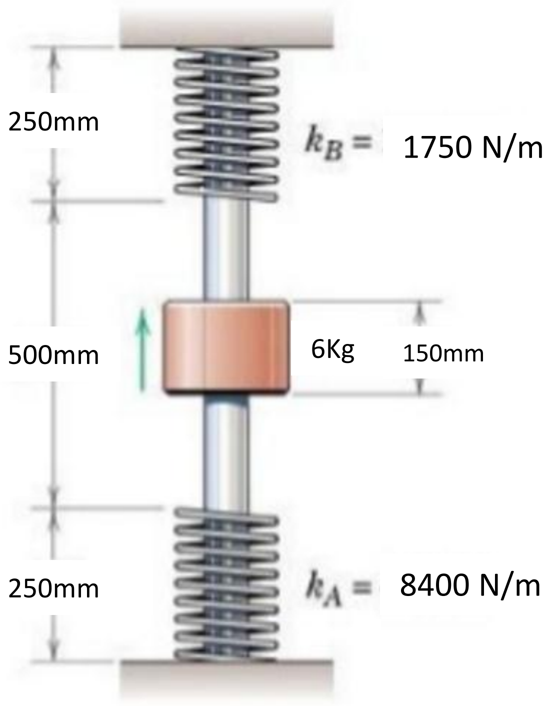
$P = FV = 45.6\,(2.22)$

$P = 101.4$ W

Problem 3/150 :

The springs are undeformed in the position shown . If the ***6 Kg*** collar is released from rest in the position where the lowest spring is compressed ***125 mm*** .

Determine the maximum compression ***X_B*** of the upper spring .



Problem 3/162

Solution :

∑ stablish datum at release point

$T_A + V_A = T_B + V_B$;

$0 + 0.5(K_A.X_A^2) = 0 + 0.5(K_B.X_B^2) + mg(X_A+d+X_B)$;

$0.5(8500)(0.125)^2 = 0.5(1750)X_B^2 + 6(9.81)(0.125+0.5-0.15+X_B)$

$\mathbf{X_B}$ = 0.1766m = **176.6 mm**

(The collar moves a distance of 0.5 – 0.15 = 0.35 m)

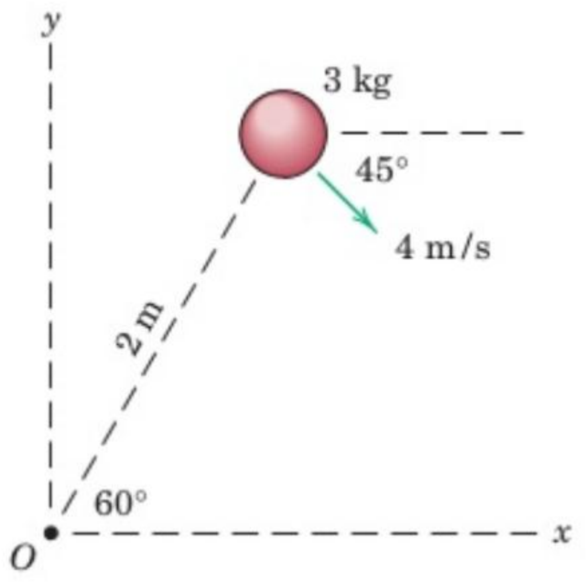
Problem 3/216 :

The ***3 Kg*** sphere moves in the **x-y plane** and has the indicated velocity at a particular instant .

Determine its:- **(a)** linear moment .

(b) angular momentum about point ***O***.

(c) kinetic energy .



Problem 3/216

Solution :

(a) $\vec{G} = m\vec{v}$

$= 3.4 (\cos(45°)i + \sin(45°)j)$

$\mathbf{= 8.49i - 8.49j\ Kg.}$

(b) $\vec{H_o} = \vec{r} * m\vec{v} = \vec{r} * \vec{G}$

$= 2(\cos(60°)i + \sin(60°)j) \times (8.49i - 8.49j)$

$= 2[-4.25k - 7.35k]$

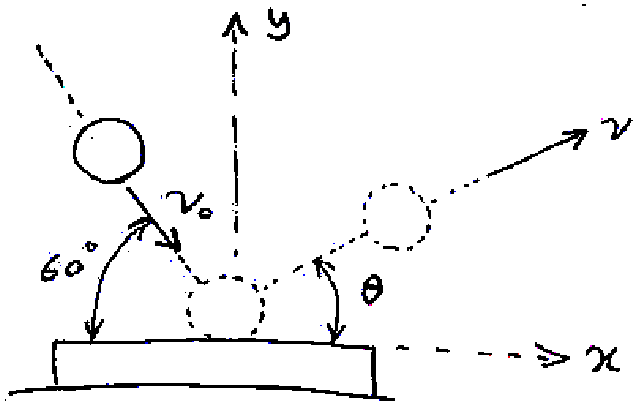
$\mathbf{= -23.2\ Kg.m^2/s}$

(c) $T = 0.5\ m v^2 = 0.5(3)(4^2)$

$\mathbf{= 24\ J}$

*Problem 3/250 *

The steel ball strikes the heavy steel plate with a velocity v.= 24 m/s at angle of 60* with the horizontal . If the coefficient of restitution is e=0.8 . Compute the velocity v and its direction 0 with which the ball rebounds from the plate .



Solution

*during impact $\sum fx = 0$ so no change in X velocity .

*component.

$$V(cos0) = 24(cos60) = 24(0.5) = 12m/s$$

In y - diriction

$$E = \frac{V2' - V1'}{V2 - V1} = \frac{0 - v * sin\Theta}{-v * cos30 - 0} = 0.8 = \frac{v * sin\Theta}{v * cos30}$$

$$0.8 = \frac{v * sin\Theta}{24 * cos30} >>> v * sin\Theta = 16.63 >>> v * cos\Theta = 12$$

$$\frac{v * sin\Theta}{v * cos\Theta} = \frac{16.63}{12} = tan\Theta = 1.39 >>> \Theta = 54.2$$

$$V = \frac{12}{cos\Theta} = \frac{12}{cos54.2} = 20.5m/s$$

#chapter 4

(Kinetics of systems of particles)

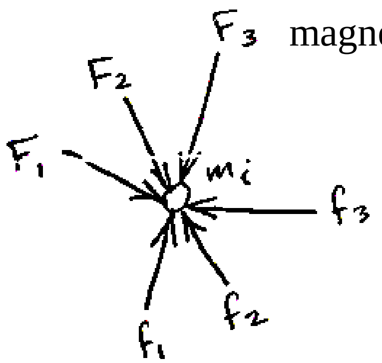
*the principles applied to a single particle will be extended to a system of participles.

*A rigid body is defined as a solid system of particles , where in the distance between participles remain unchanged .

-example of rigid-body problems:
Machines , land & aircraft , rockets , and space-craft.

*A non-rigid body could be a solid body which changes shape with time due to deformation . It could also be a liquid or gas.

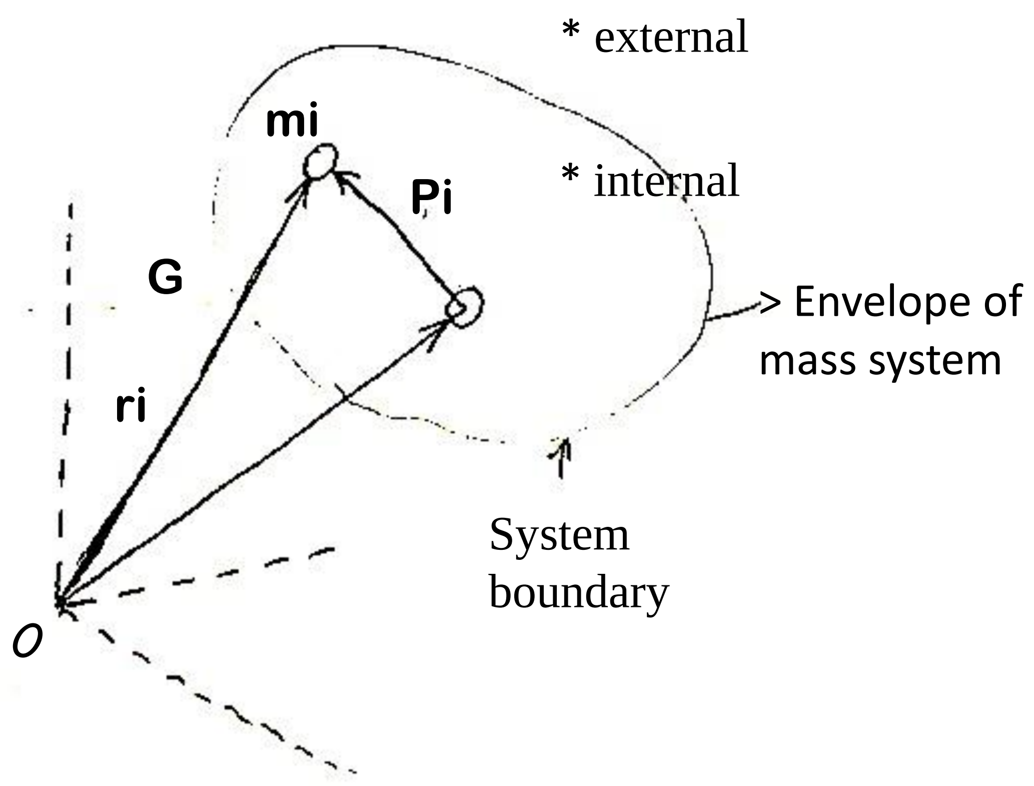
**Generalized Newton's 2nd law



F3 external forces : resulting from external bodies ,gravitational , electric , or magnetic .

f3 internal forces (reactions)

M is center of mass of the participles



$$m * r = \sum mi * ri$$

where

$$m = \sum mi$$

*total system mass they are n particles.

Applying Newton's 2nd law to the system :

F1 + F2 + F3 + ………… + f1 + f2 + f3 + …….. $= \sum m_i \cdot \ddot{r}_i$ ⟵ Acceleration of mi

For all particles :

$$\sum F + \sum f = \sum m_i \cdot \ddot{r}_i$$

External: $\sum F$; Internal: $\sum f$ = 0 (internal forces cancel out)

Since $m\bar{r} = \sum m_i \cdot r_i$ ⟹ $m\ddot{\bar{r}} = \sum m_i \cdot \ddot{r}_i$

- m: Total mass
- $\bar{r}$: Position vector of the center of mass
- m_i: Mass of particle
- r_i: Position vector of particle

⟹

$$\boxed{\sum F = m\ddot{\bar{r}} = m\bar{a}}$$

→ Generalized Newton's 2nd law of motion for a mass system . Or equation of motion of m . Or principle of motion of the mass center

$\bar{a}$: Acceleration of the center of mass of the system

Component form:

$$\sum F_x = m\bar{a}_x \qquad \sum F_y = m\bar{a}_y \qquad \sum F_z = m\bar{a}_z$$

$\sum F$ **:** Generally does not pass through G.

Work – Energy :

m_i : $(U_{1-2})_i$ **=** ΔT_i ⟶ ${}^1/_2\, m_i v_i^2$

Work done on mi by external forces only

For the entire system : $\sum_{i=1}^{n}(U_{1-2})_i = \sum_{i=1}^{n} \Delta T_i$

Work done by internal forces is zero because it cancels out .
If gravity and elastic energy is included [non-rigid body]:

$$U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

U_{1-2}: work done on a non-rigid system.

ΔT :kinetic energy .

ΔV_g: gravitational potential energy

ΔV_e: elastic potential energy .

$$\underbrace{\Delta T + \Delta V_g + \Delta V_e}_{\Delta E}$$

ΔE:chang in mechanical energy .

OR

$$U'_{1-2} + T_1 + V_{g_1} + V_{e_1} = T_2 + V_{g2} + V_{e2}$$

Kinetic energy revisited :
From relative motion:

$$v_i = \bar{v} + \dot{p}i$$

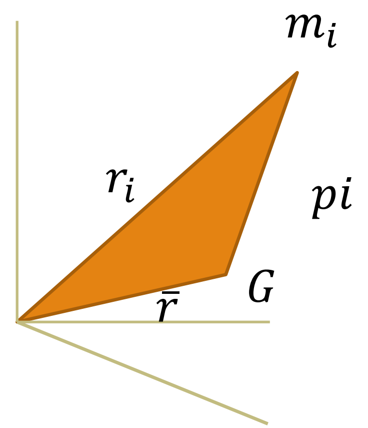
$\bar{v}$: velocity of mass center

pi: velocity w. r. t G

since $v_i^2 = v_i . v_i$

$$T = \sum \frac{1}{2} m_i v_i^2$$

$$T = \sum \frac{1}{2} m_i v_i . v_i = \sum \frac{1}{2} m_i (\bar{v} + \dot{p}i)(\bar{v} + \dot{p}i)$$



$$T = \sum \frac{1}{2} m_i \bar{v}^2 + \sum \frac{1}{2} m_i |\dot{p}i|^2 + \sum m_i \bar{v}\, \dot{p}i$$

$$= \bar{v} \sum m_i\, \dot{p}i = \bar{v} \frac{d}{dt} \sum m_i\; pi = 0$$

$\sum m_i\, pi = 0$: measured form mass center

$$T = \frac{1}{2} \bar{v}^2 \sum m_i + \sum \frac{1}{2} m_i |\dot{p}i|^2$$

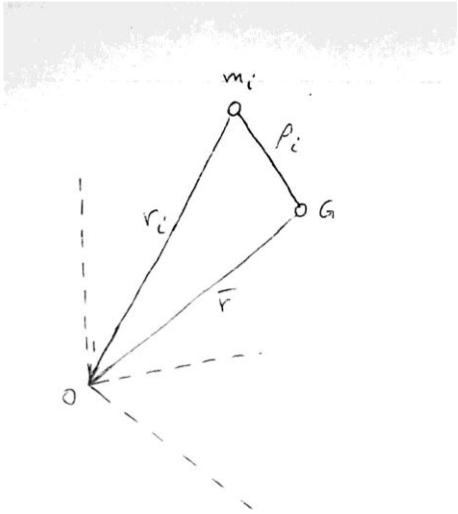
$$T = \frac{1}{2} m\bar{v}^2 + \sum \frac{1}{2} m_i |\dot{p}i|^2$$

$\frac{1}{2} m\bar{v}^2$: T of the mass center G

$\sum \frac{1}{2} m_i |\dot{p}i|^2$: energy of particles relative to mass center.

Impulse-Momentum

(a) Linear momentum (G)



$$Gi = mivi \leftarrow = \dot{r}i$$

$$G = \sum mi\, vi$$

$$vi = \overline{v} + \dot{p}i$$

$$\sum mi\, pi = m\overline{p} = 0$$

$$G = \sum mi\,(\overline{v} + \dot{p}i) = \sum mi\,\overline{v} + \frac{\mathrm{d}\sum \mathrm{mi\ pi}}{\mathrm{d}t}$$

$$= \overline{v}\,\sum mi + \frac{\mathrm{d}(0)}{\mathrm{d}t} = m\,\overline{v} \Rightarrow \boxed{G = m\,\overline{v}}$$

velocity of center of mass G

total mass

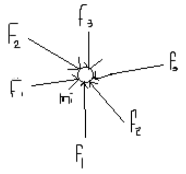
$$\dot{G} = m\dot{\overline{v}} = m\overline{a} = \sum F \Rightarrow \sum F = \dot{G}$$

time rate of change of G of the system

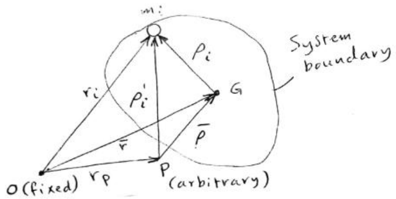
resultant force on mass system

(b) Angular Momentum (Ho)

Ho about O:



$$ap = \ddot{r}p$$



$$Ho = \sum (ri \times mi\,vi) \Rightarrow \dot{H}o = \sum (\dot{r}i \times mi\,vi) + \sum (ri \times mi\dot{v}i)$$

$$= 0 + \sum (ri \times Fi) \Rightarrow$$

$$(= vi) \Rightarrow vi \times vi = 0$$

$$\boxed{\sum Mo = Ho}$$

→ for external forces, ∑Mo of internal forces cancel out

Ho about G (mass center) → H_G

$$HG = \sum pi \times mi\,\dot{ri}$$

$$\left[\; ri = \bar{r} + pi \Rightarrow \dot{ri} = \dot{\bar{r}} + \dot{pi} \;\right]$$

$$HG = \sum pi \times mi\,(\dot{\bar{r}} + \dot{pi})$$

$$HG = \boxed{\sum pi \times mi\,\dot{\bar{r}}} + \sum pi \times mi\,\dot{pi}$$

$$\left[\; -\dot{\bar{r}} \times \boxed{\sum mi\,pi} = 0 \;\right]$$

[By definition of the mass center G]

$$HG = \sum pi \times mi\,\dot{pi}$$

Absolute angular momentum because $\dot{ri}$ is used.

Relative angular momentum because $\dot{pi}$ is used.

Since

$$HG = \sum pi \times mi\,\dot{pi} \;\Rightarrow\; \dot{HG} = \sum \dot{pi} \times mi\,(\dot{\bar{r}} + \dot{pi}) + \sum pi \times mi\,\ddot{ri}$$

$$= \sum \dot{pi} \times mi\,\dot{\bar{r}} + \sum \dot{pi} \times mi\,\dot{pi}$$

Parallel=0

$$-\dot{\bar{r}} \times \sum mi\,\dot{pi} = -\dot{\bar{r}} \times \frac{\mathrm{d} \sum \mathrm{mi\,pi}}{\mathrm{d}t} = 0 \longrightarrow$$

$$\Rightarrow \dot{HG} = \sum pi \times mi\,\ddot{ri} = \sum pi \times mi\,ai = \sum pi \times (Fi + fi)$$

sum of internal moments is zero

$$\dot{HG} = \sum pi \times Fi = \sum MG$$

external moments

$$\Rightarrow \boxed{\sum MG = \dot{HG}}$$

Good for rigid & non-rigid systems

H_0 about P (arbitrary point)

$$H_P = \sum P_i' \times m_i \dot{r}_i = \sum (\bar{P} + P_i) \times m_i \dot{r}_i$$

$$H_P = \bar{P} \times \sum m_i \dot{r}_i + \sum P_i \times m_i \dot{r}_i$$

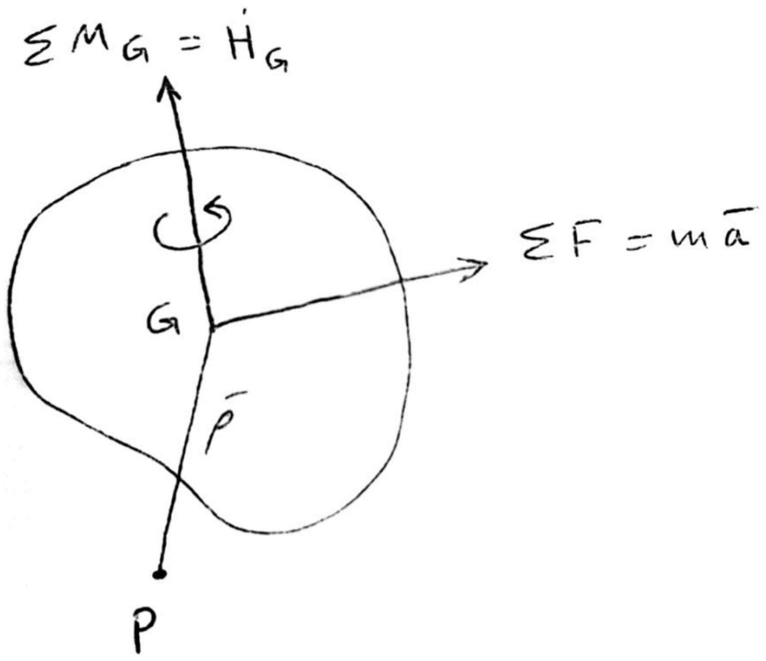
$$H_p = \bar{P} \times \sum m_i v_i + H_G = \bar{P} \times m\bar{v} + H_G$$

$$\Rightarrow H_p = H_G + \bar{P} \times m\bar{v}$$

- $H_G = \sum P_i \times m_i \dot{r}_i$

$$\sum M_P = \sum M_G + \bar{P} \times \sum F$$

$$\Rightarrow \sum M_P = \dot{H}_G + \bar{P} \times m\bar{a}$$



- When a point P whose acceleration is known is used as a moment center ;

$$\sum M_P = (\dot{H}_P)_{relative} + \overline{P} \times m a_P$$

$$\Rightarrow \sum M_P = (\dot{H}_P)_{rel}$$ If :- 1. $a_P = 0$

2. $\overline{P} = 0$

3. $\overline{P}$ and a_P are parallel

Conservation of Energy and Momentum

- Conservation of Energy (for a system)

If there is no energy loss due to friction or dissipation ; Then there's No net change in Mech. Energy ($\Delta E=0$)

$\Rightarrow$ ΔT+ΔVg+ΔVe=0 or

T1 + Vg1 + Ve1 = T2 +Vg2 + Ve2 $\Rightarrow$ (no work U1 - U2 = 0)

Law of conservation of dynamical energy

(b) conservation of Momentum (for a system)

Since $\sum F = \dot{G}$ if $\sum F = 0$ $\Rightarrow$ $\dot{G}$ =0 $\Rightarrow$

G1 = G2 → principle of conservation of Linear Momentum for a mass system (no linear Impulse)

since $\sum M0 = \dot{H}0$ if $\sum M0 = 0$ → $\dot{H}0 = 0$

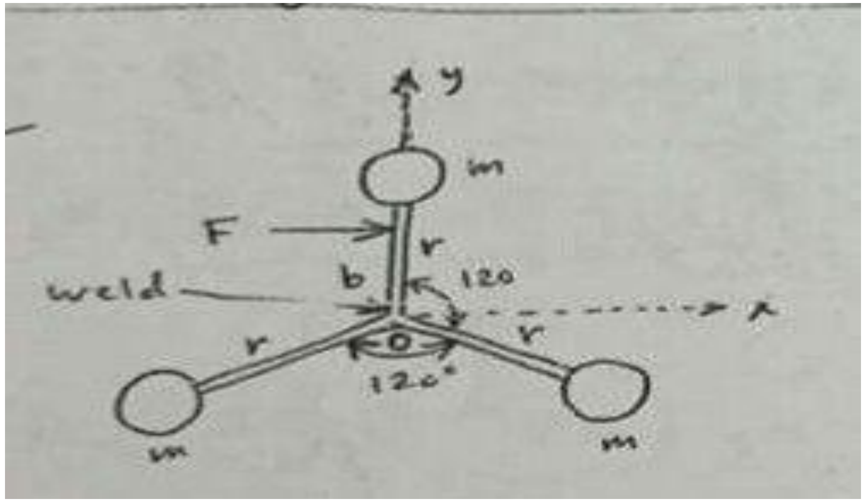
(H0)1=(H0)2 or (HG)1 =(HG)2 →

Principle of conservation of angular moment for General mass system (no angular Impulse)

EX

Rigid equiangular frame of negligible mass , resting on a horizontal surface

F is suddenly applied



Find

A- a0

B- $\ddot{\theta}$

Solu

a) $\sum F = m\bar{a}$ ➡ F1= 3m $\bar{a}$ ➡ $\bar{a}$ = a0 = $\frac{F}{3m}\hat{\imath}$

b) v = r $\dot{\theta}$ H0= HG = 3r m v = 3 r m (r $\dot{\theta}$) =3 m $r^2\dot{\theta}$

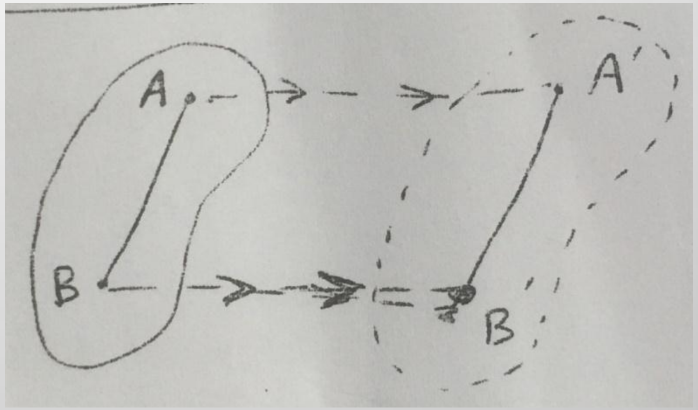
$$\sum MG = \dot{H}G \rightarrow Fb = \frac{d}{dt}(3mr^2\dot{\theta}) = 3mr^2\ddot{\theta} \rightarrow \ddot{\theta} \rightarrow \frac{Fb}{3m\dot{r}}$$

Chapter - 5

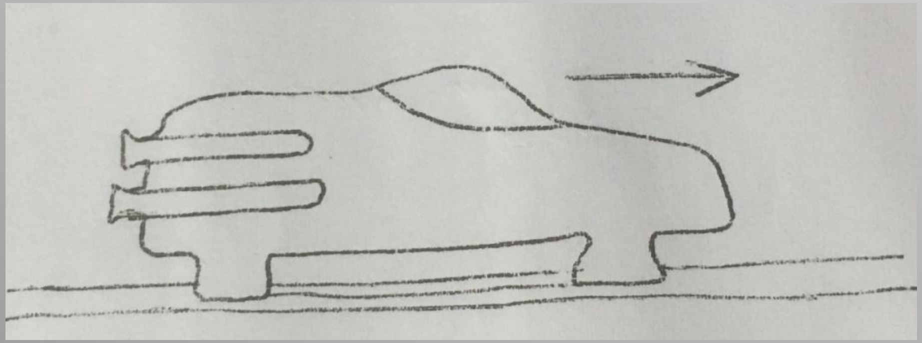
Plane kinematics of rigid bodies

Types of rigid-body plane motion

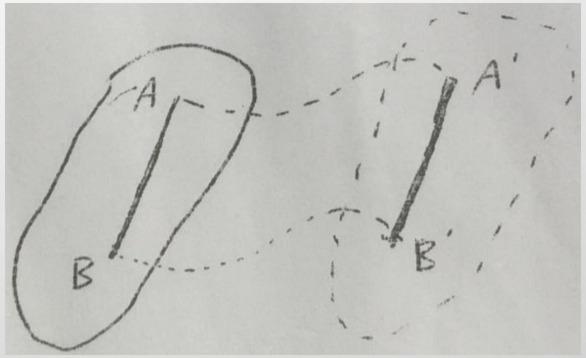
- a) Rectilinear translation :



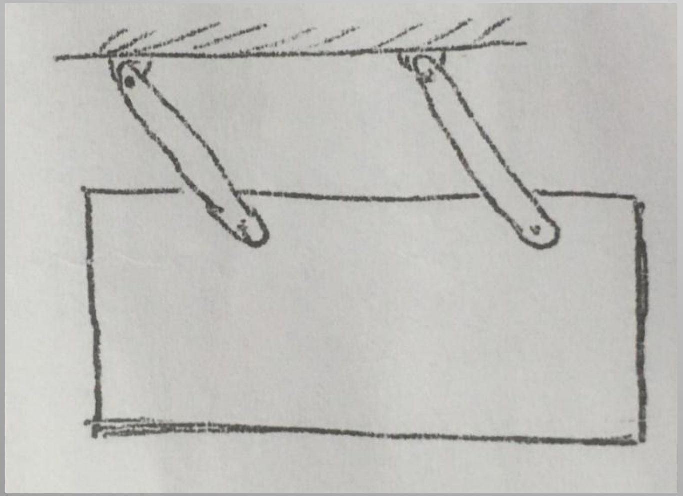
- Example : Rocket test sled



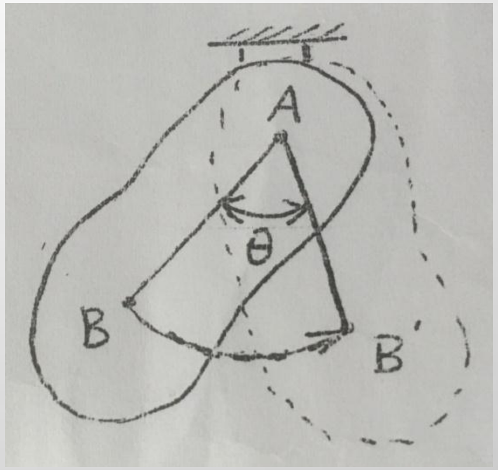
- b) Curvilinear translation :



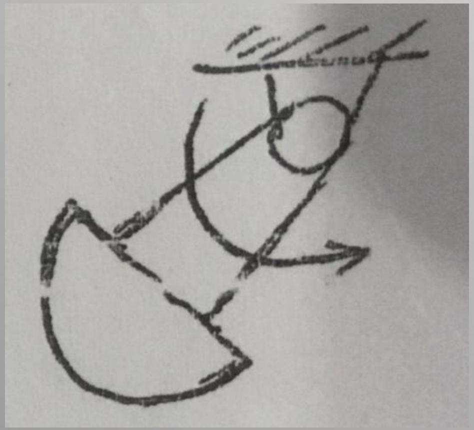
- Example : Parallel-link swinging plate



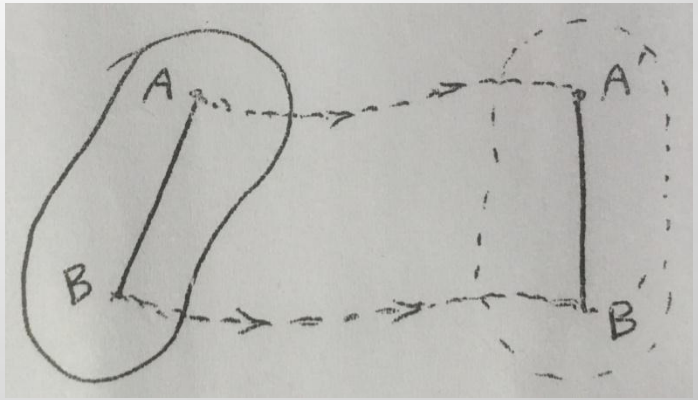
- c) Fixed-axis rotation :



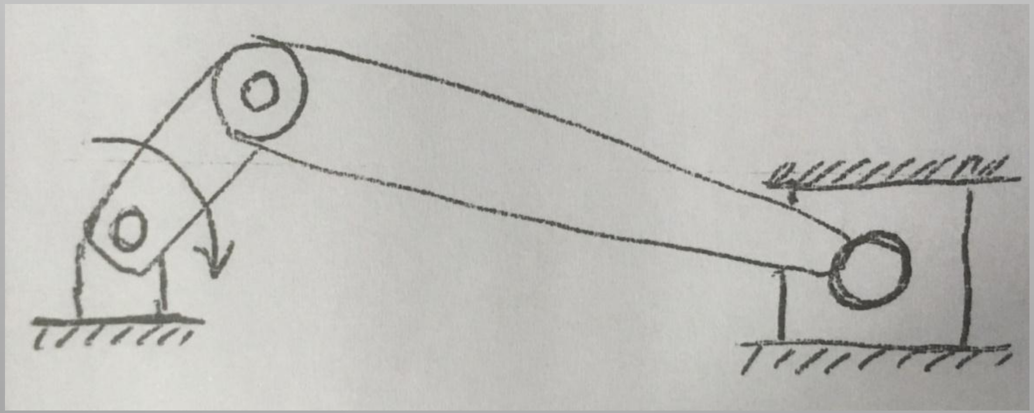
- Example : Compound pendulum



- d) General plane motion :
- translation + rotation



- Example : Connecting rod in a reciprocating engine



Rotation

- A) Angular motion relations

$$w = \frac{d\theta}{dt} = \dot{\theta}$$

$$\alpha = \frac{dw}{dt} = \dot{w}$$

$$\alpha = \frac{d}{dt}(\frac{d\theta}{dt}) = \frac{d^2\theta}{dt^2} = \ddot{\theta}$$

$$w = \frac{d\theta}{dt} \Rightarrow dt = \frac{d\theta}{w}$$

$$\alpha = \frac{dw}{dt} \Rightarrow dt = \frac{dw}{\alpha} \Rightarrow \frac{d\theta}{w} = \frac{dw}{\alpha}$$

$$wdw = \alpha d\theta \Rightarrow \dot{\theta} d\dot{\theta} = \ddot{\theta} d\theta$$

- For constant α :

$$w = w_o + \alpha t$$

$$w^2 = w_o^2 + 2\alpha(\theta - \theta_o)$$ at t=0

$$\theta = \theta_o + w_o t + \frac{1}{2}\alpha t^2$$

- B) Rotation about a fixed axis

$$v = rw$$

$$a_n \frac{v^2}{r} = \frac{r^2 w^2}{r} = rw^2 = vw$$

$$a_t = r\alpha$$

- In vector form :

$$\vec{v} = \vec{w} \times \vec{r}$$

$$\vec{a}_n = \vec{w} \times (\vec{w} \times \vec{r})$$

$$\vec{a}_t = \vec{\alpha} \times \vec{r}$$

Example :-

- A right-angled bar ; if :

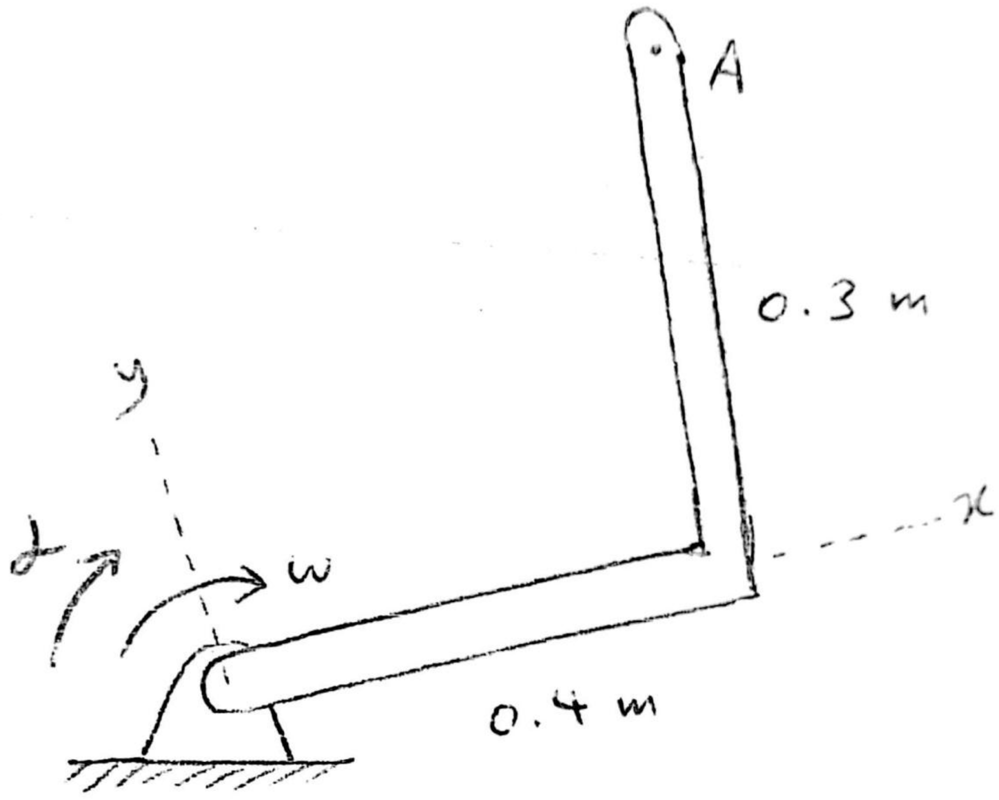
  $\alpha = -4\, rad/s^2$

  $\omega = 2\, rad/s$

- Find :-

  $v_A = ?$

  $\vec{a}_A = ?$



Solution :-

- $\vec{\omega} = -2\hat{k}\, rad/s$
- $\vec{\alpha} = -4\, rad/s^2 \left(-\hat{k}\right)$

$\vec{\alpha} = 4\hat{k}\, rad/s^2$

- $\vec{v} = \vec{\omega} \times \vec{r}$

$\vec{v} = -2\hat{k} \times \left(0.4\hat{i} + 0.3\hat{j}\right) = 0.6\hat{i} - 0.8\hat{j}\; m/s$

- $\vec{a}_A = \vec{a}_n + \vec{a}_t$
- $\vec{a}_n = \vec{\omega} \times (\vec{\omega} \times \vec{r})$

$$\vec{a}_n = -\hat{k} \times (0.6\hat{i} - 0.8\hat{j}) = -1.2\hat{i} + 1.6\hat{j}\, m/s^2$$

- $\vec{a}_t = \vec{\alpha} \times \vec{r}$

$$\vec{a}_t = 4\hat{k} \times (0.4\hat{i} + 0.3\hat{j}) = -1.2\hat{i} + 1.6\hat{j}\, m/s^2$$

$$\Rightarrow \vec{a}_A = -2.8\hat{i} + 0.4\hat{j}\, m/s^2$$

$$\Rightarrow |v| = \sqrt{(0.6)^2 + (0.8)^2} = 1 m/s$$

$$\Rightarrow |a| = \sqrt{(2.8)^2 + (0.4)^2} = 2.83 m/s^2$$

Absolute Motion

The use of geometric relations which define the configuration of the body to derive velocities and acceleration.

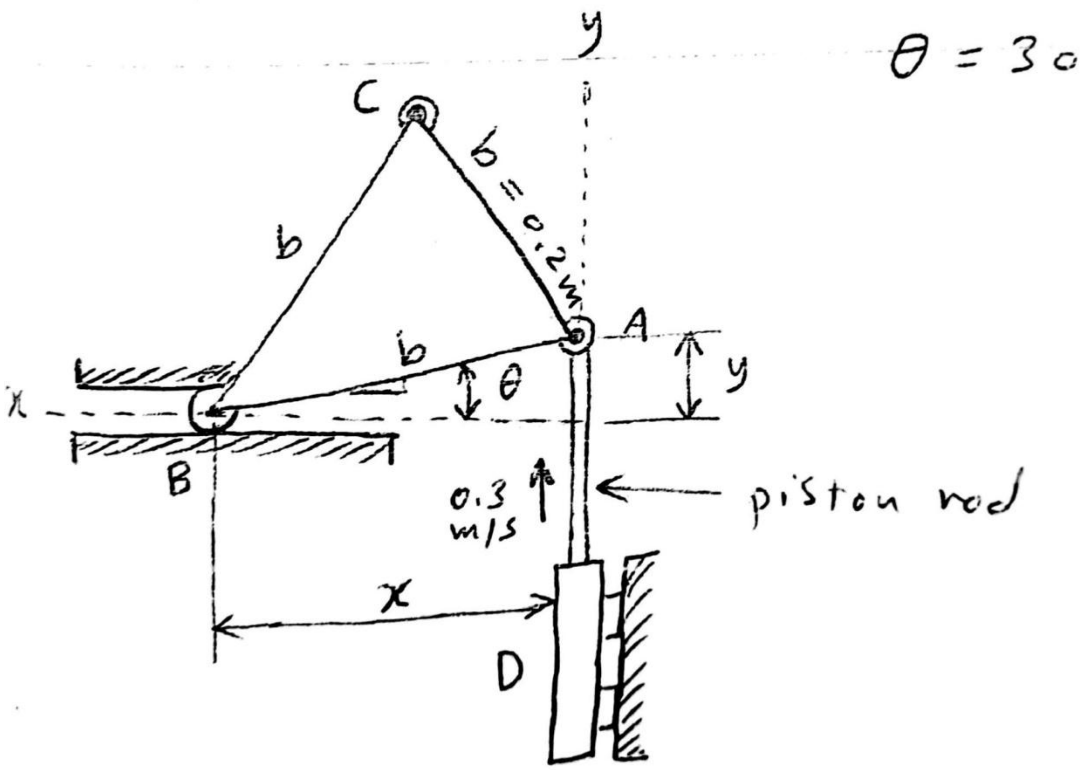
Example :-

Equilateral triangular plate ABC is controlled by hydraulic cylinder D.

Find :-

1. v and a of the center of B
2. ω and α of the edge CB

Solution:-



$$v_A = \dot{y} = 0.3\,m/s$$

$$a_A = \ddot{y} = 0$$

From the geometry;

$$x^2+y^2=b^2$$

$$\frac{d}{dt}(x^2+y^2=b^2)=$$

$$2\dot{x}x+2y\dot{y}=0$$

$$\Rightarrow \dot{x}x+y\dot{y}=0$$

$$\Rightarrow \dot{x}=\frac{-y\dot{y}}{x}=\frac{-y}{x}\dot{y}$$

$$\frac{d}{dt}(x\dot{x}+y\dot{y}=0)=x\ddot{x}+\dot{x}\dot{x}+y\ddot{y}+\dot{y}\dot{y}=x\ddot{x}+\dot{x}^2+y\ddot{y}+\dot{y}^2=0$$

$$\Rightarrow \ddot{x}=\frac{-\dot{x}^2-y\ddot{y}-\dot{y}^2}{x}=-\frac{\dot{x}^2+\dot{y}^2}{x}-\frac{y}{x}\ddot{y}$$

But

$$y = b\sin\theta$$

$$x = b\cos\theta$$

$$\ddot{y} = 0$$

$$\Rightarrow v_B = \dot{x} = -\frac{y}{x}\dot{y} = \frac{-b\sin\theta}{b\cos\theta}v_A$$

$$\Rightarrow v_B = -v_A\tan\theta$$

$$\Rightarrow a_B = \ddot{x} = \frac{-(-v_A \tan\theta)^2 - 0 - v_A^2}{b\cos\theta} = \frac{-v_A^2 \tan^2\theta - v_A^2}{b\cos\theta}$$

$$\Rightarrow a_B = \frac{-v_A^2(\tan^2\theta + 1)}{b\cos\theta} = \frac{-v_A^2(\sec^2\theta)}{b\cos\theta} = \frac{-v_A^2 \sec^3\theta}{b}$$

with

$$v_A = 0.3\, m/s$$

and

$$\theta = 30^\circ$$

$$v_A = -0.3\tan(30) = -0.3(\frac{1}{\sqrt{3}}) = -0.173 m/s(\rightarrow)$$

$$a_B = \frac{-(0.3)^2 \sec^3(30)}{0.2} = -0.693 m/s^2(\rightarrow)$$

To find angular motion of CB , differentiate θ ;

$$y = b\sin(\theta) \Rightarrow y^{\bullet} = b(\cos\theta)\theta^{\bullet} \Rightarrow$$

$$\theta^{\bullet} = w = \frac{y^{\bullet}}{b\cos(\theta)} = \frac{v_A}{b}\sec\theta = \frac{0.3}{0.2}\sec(30) = 1.73 rad/s(ccw)$$

$$\alpha = w^{\bullet} = \frac{v_A}{b}\sec\theta^{\bullet}\tan\theta = \frac{v_A}{b}\sec\theta\tan\theta[\frac{v_A}{b}\sec\theta]$$

$$\alpha = \frac{{v_A}^2}{b^2}\sec^2\theta\tan\theta = \frac{0.3^2}{0.2^2}\sec^2(30)\tan(30) =$$

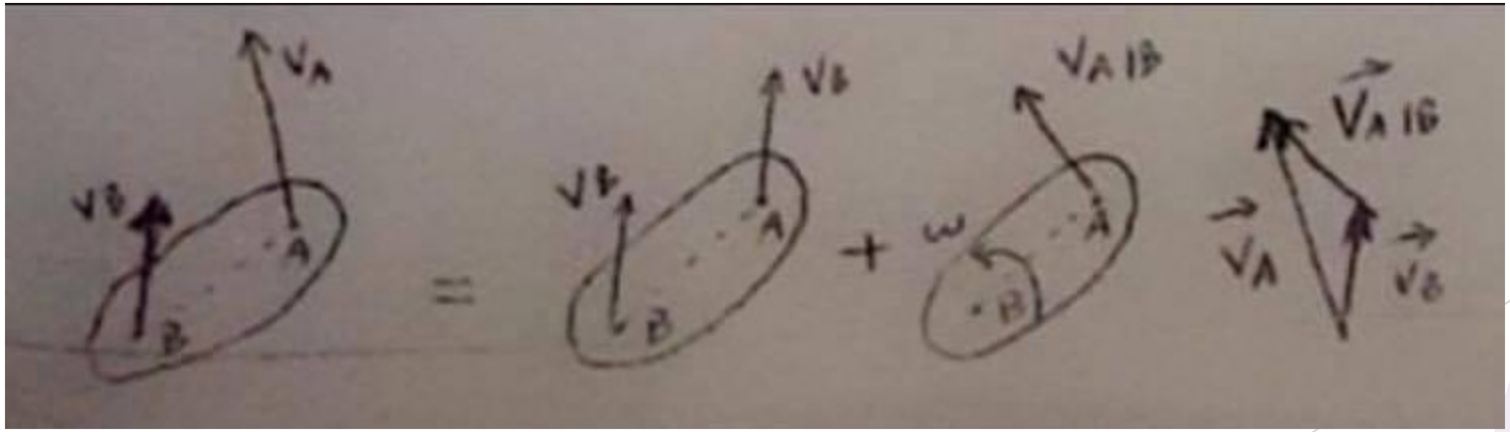
$$\frac{0.3^2}{0.2^2}(\frac{2}{\sqrt{3}})\frac{1}{\sqrt{3}} = 1.73 rad/s^2(ccw)$$

Relative Velocity (of rigid body)

$$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B} \qquad \text{OR} \qquad \vec{V}_{A/B} = \vec{V}_A - \vec{V}_B$$

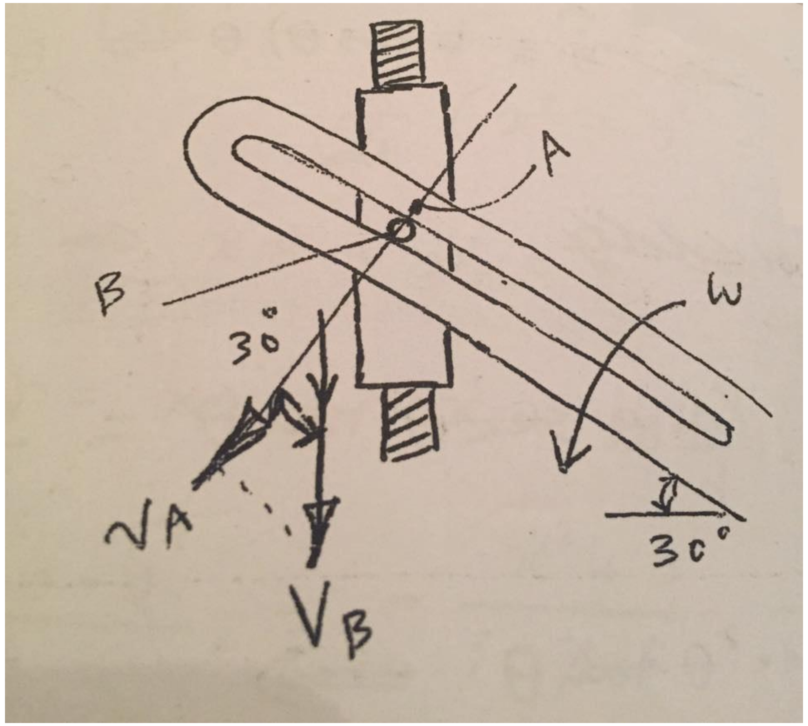
For a rigid body , $\vec{V}_{A/B} = rw$

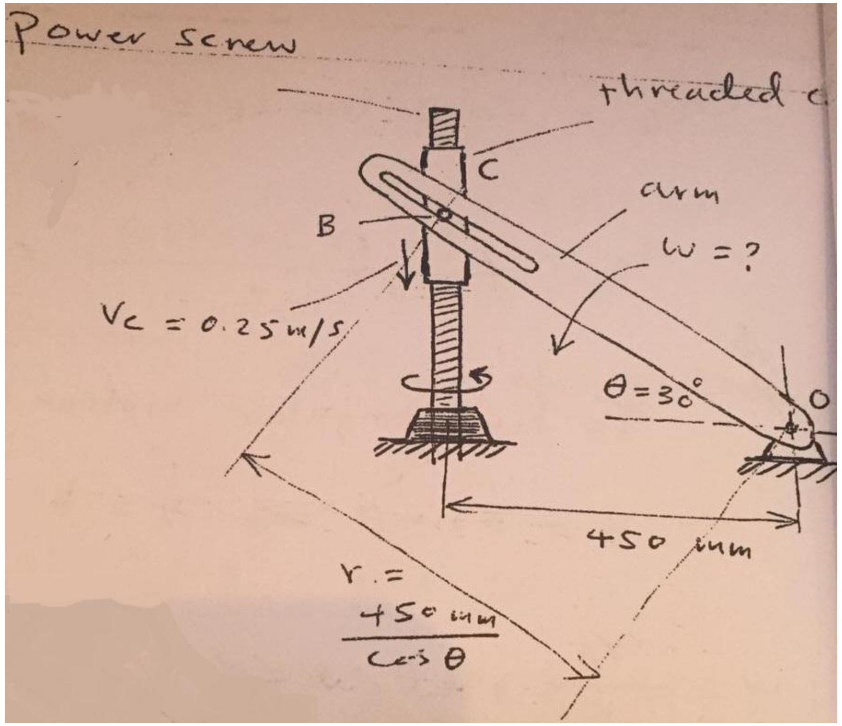
In vector form $\vec{V}_{A/B} = \vec{r} \times \vec{w}$



Power screw

<u>EX:</u> The power screw gives the collar C a velocity of V_C=0.25m/s find w of the arm when θ=30°.

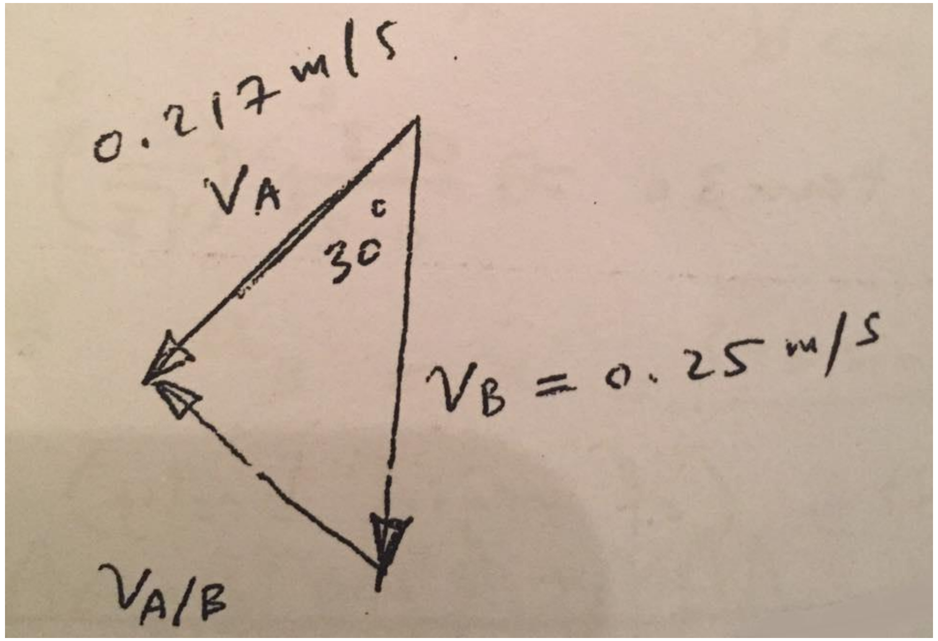




$V_B = V_c$

$V_A = V_B \cos\theta = 0.25 \cos 30° = 0.217$ m/s

$V = wr \Longrightarrow w = v/r = \dfrac{V_A = 0.217}{0.45/\cos 30} = 0.417$ rad/ccw



Instantaneous Center of zero velocity (ICZV)

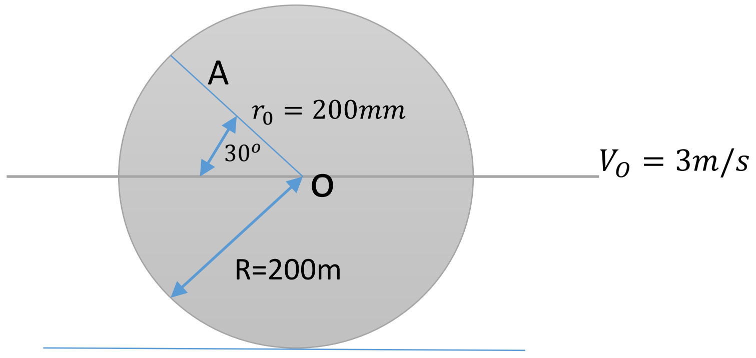
It is a unique reference point which momentarily has a zero velocity.

$V_A = w\, r_A$ ⇓ $w = V_A / r_A$

$V_B = w\, r_B$ ⇓ $V_B = (V_A / r_A)\, r_B$ ⇓ $V_B = (r_B / r_A)\, V_A$

EX .. The wheel rolls to the right without slipping. Locate the 1 czv ? find V_A ?

- Solu:



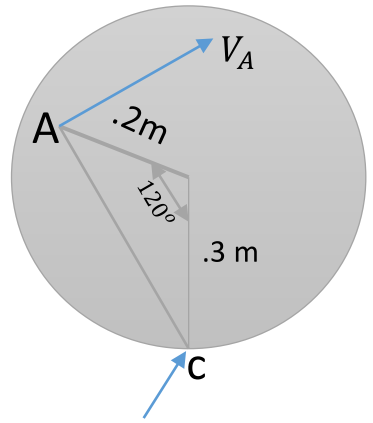
$$v = wr \rightarrow w = \frac{v}{r}$$

$$w = \frac{v_o}{\vec{oc}} = \frac{3}{0.3} = 10rad/s$$

$$\vec{AC} = \sqrt{0.3^2 + 0.2^2 - 2(0.3)(0.2)COS120}$$

$$\vec{AC} = 0.436m$$

$$\rightarrow V_A = W\,\vec{AC} = (10)(0.436) = 4.36\,m/s$$



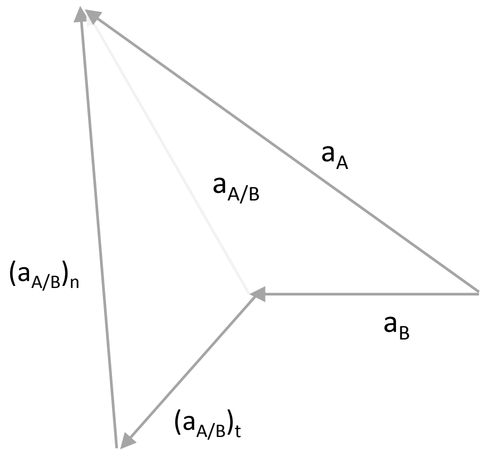
- Relative Acceleration (of a rigid body)

$a_A = a_B + a_{A/B} = a_B + (a_{A/B})_n + (a_{A/B})_t$

- $(a_{A/B})_n = (V_{A/B})^2 = rw^2$
- $(a_{A/B})_t = (V_{A/B})_t = r\alpha$

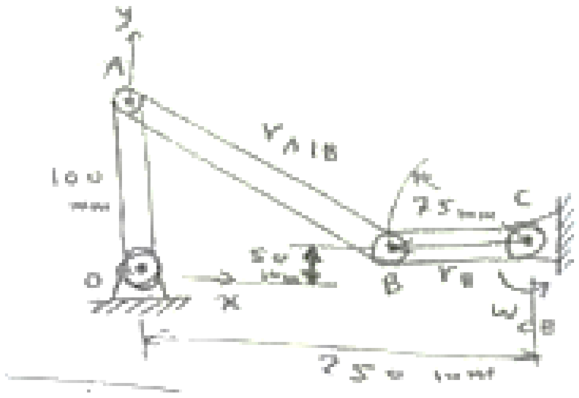
In vector form :

- $(a_{A/B})_n = \vec{w} * (\vec{w} * \vec{r})$
- $(a_{A/B})_t = \vec{\alpha} * \vec{r}$



EX:

$W_{CB} = 2\frac{rad}{s}, W_{AB} = -\frac{6}{7}\frac{rad}{s}, W_{OA} = -\frac{3}{7}\frac{rad}{s}, \alpha_{AB} = ?, \alpha_{OA} = ?$



Solution:

$$\overrightarrow{\alpha_A} = \overrightarrow{\alpha_B} + (\overrightarrow{\alpha_{A/B}})_n + (\overrightarrow{\alpha_{A/B}})_t$$

$$\overrightarrow{\alpha_A} = \alpha_{OA} \times r_A + W_{OA} \times (W_{OA} \times r_A)$$

$$= \alpha_{OA}\,\hat{k} \times 100\hat{j} + \left(-\frac{3}{7}\hat{k}\right) \times \left(-\frac{3}{9}\hat{k} \times 100\hat{j}\right) = -100\,\alpha_{OA}\,\hat{\imath} - 100\left(\frac{3}{7}\right)^2 \hat{j}\ mm/s^2$$

$$\overrightarrow{\alpha_B} = \alpha_{CB} \times \overrightarrow{r_B} + \overrightarrow{W_{CB}} \times \left(\overrightarrow{W_{CB}} \times \overrightarrow{r_B}\right) = 0 + 2\,\hat{k} \times \left(2\,\hat{k} \times (-75\hat{\imath})\right) = 300\hat{\imath}\frac{mm}{s^2}$$

$$\left(\overrightarrow{\alpha_{\frac{A}{B}}}\right)_n = W_{AB} \times (W_{AB} \times r_{\frac{A}{B}}) = -\frac{6}{7}\hat{k} \times \left(\left(-\frac{6}{7}\hat{k}\right) \times (-175\hat{\imath} + 50\hat{j})\right) = \left(\frac{6}{7}\right)^2 (175\hat{\imath} - 50\hat{j})\frac{mm}{s^2}$$

$$\left(\overrightarrow{\alpha_{\frac{A}{B}}}\right)_t = \alpha_{AB} \times r_{\frac{A}{B}} = \alpha_{AB}\hat{k} \times (-175\hat{\imath} + 50\hat{j}) = -50\alpha_{AB}\hat{\imath} - 175\alpha_{AB}\,\hat{j}\ \frac{mm}{s^2}$$

$$-100\alpha_{OA} = 429 - 50\alpha_{AB}$$

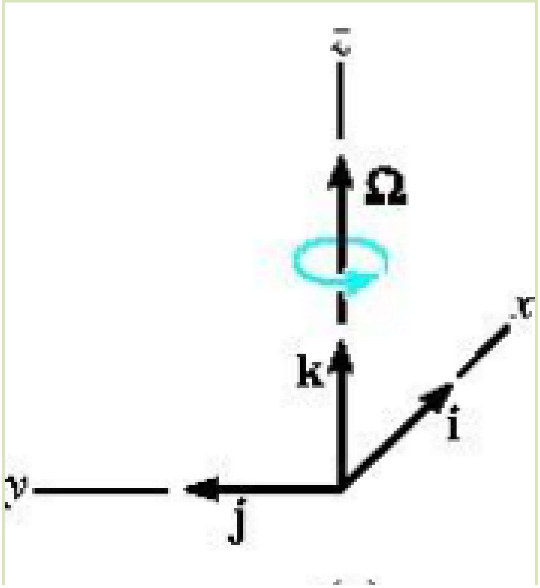
$$-18.37 = -36.7 - 175\alpha_{AB} \ \rightarrow$$

$$\alpha_{AB} = -0.1050\ \frac{rad}{S^2}(cw\ DIRECTION\ (-\hat{k}))\ \&\ \alpha_{OA} = -4.34\frac{rad}{S^2}$$

Motion Relative to Rotating Axes:

$$w = \dot{\theta} k = \frac{d\theta}{dt} \quad ,, \quad w\,dt = d\Theta$$

(dî) Is small change in î ,
[dî = dΘ ĵ]



$$\frac{di}{dt} = \frac{d\theta}{dt}\,\hat{j} \rightarrow \dot{\hat{i}} = \dot{\theta}\hat{j} = w\hat{j}$$

$\dot{\hat{i}} = \omega \hat{j}$ and $\dot{\hat{j}} = -\omega \hat{i}$

Or

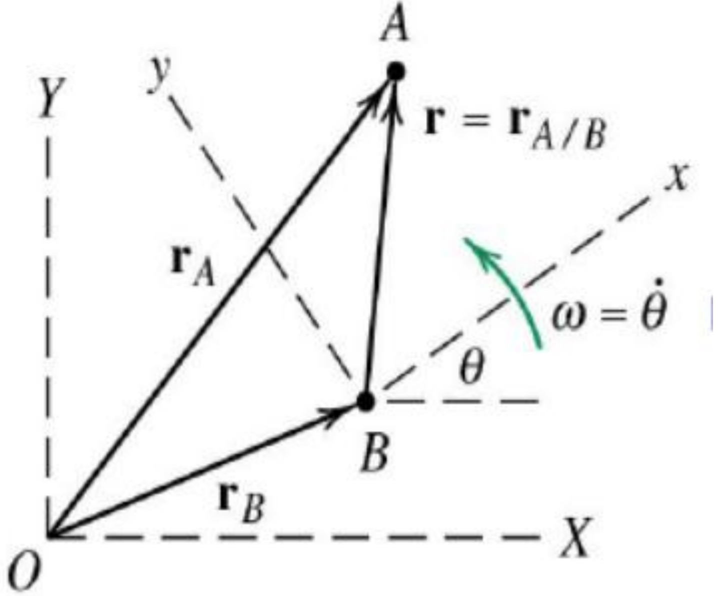
$\dot{\hat{i}} = \vec{\omega} \times \hat{i}$ *and* $\dot{\hat{j}} = \vec{\omega} \times \hat{j}$

W x î = w ĵ
W x ĵ = - w î

→ Î = w ĵ = w x î

$\dot{\hat{i}} = \vec{w}\,\hat{i}$ and $\dot{\hat{j}} = \vec{w}\,\hat{j}$

Relative velocity:



$$\vec{r}_A = \vec{r}_B + \vec{r} = \vec{r}_B + (x_i + y_i)$$

$$\dot{\vec{r}}_A = \dot{\vec{r}}_B + \frac{d}{dt}(x_i + y_j)$$

$$= \dot{\vec{r}}_B + (x\dot{\hat{\imath}} + y\dot{\hat{\jmath}}) + (\dot{x}\hat{\imath} + \dot{y}\hat{\jmath})$$

$$= \dot{\vec{r}}_B + (x(\vec{w}\, X\, \vec{\hat{\imath}})) + (y(\vec{w}\, X\, \vec{\hat{\jmath}})) + (\dot{x}\hat{\imath} + \dot{y}\hat{\jmath})$$

$$= \dot{\vec{r}}_B + \vec{w}\, X\, (x_i + y_i) + \overrightarrow{v_{rel}}$$

$$\dot{\vec{r}}_A = \dot{\vec{r}}_B + \vec{w}\, X\, \vec{r} + \overrightarrow{v_{rel}}$$

- $\overrightarrow{v_{rel}}$: velocity of A relative to the rotating axes

$$\dot{\vec{v}}_A = \dot{\vec{v}}_B + \vec{w}\, X\, \vec{r} + \overrightarrow{v_{rel}}$$ → Transformation of the time derivative of the position vector between rotating and non rotating axes

↓

This equation can be generalized for any vector quantity (V)

- Transformation of a time derivative: $$\left(\frac{d\vec{v}}{dt}\right)_{xy} = \underbrace{\left(\frac{d\vec{v}}{dt}\right)_{xy}}_{\underbrace{(\dot{v}_x\hat{\imath} + \dot{v}_y\hat{\jmath})}_{\overrightarrow{v_{rel}}} + (v_x\dot{\hat{\imath}} + v_y\dot{\hat{\jmath}})} + \vec{w}\, X\, \vec{v}$$

Relative Acceleration

$$\vec{v}_A = \vec{v}_B + \vec{w} \times \vec{r} + \vec{v}_{rel}$$

$$\dot{\vec{v}}_A = \dot{\vec{v}}_B + \dot{\vec{w}} \times \vec{r} + \vec{w} \times \dot{\vec{r}} + \dot{\vec{v}}_{rel}$$

$$\vec{a}_A = \vec{a}_B + \dot{\vec{w}} \times \vec{r} + \vec{w} \times \dot{\vec{r}} + \dot{\vec{v}}_{rel}$$

Using previous relations and manipulation:

$$\vec{a}_A = \vec{a}_B + \dot{\vec{w}} \times \vec{r} + \vec{w} \times (\vec{w} \times \vec{r}) + 2\,\vec{w} \times \vec{v}_{rel} + \vec{a}_{rel}$$

- $\dot{\vec{w}} \times \vec{r}$ → a_t
- $\vec{w} \times (\vec{w} \times \vec{r})$ → a_n
- $2\,\vec{w} \times \vec{v}_{rel}$ → Coriolis acceleration
- $\vec{a}_{rel}$ → Acceleration of relative to rotates axis

General vector expression for the absolute acceleration of a particle A in terms of its acceleration $\vec{a}_{rel}$ measured relative to a moving coordinate system which rotates with an angular velocity **w** and angular acceleration $\dot{\boldsymbol{w}}$

Coriolis Acceleration

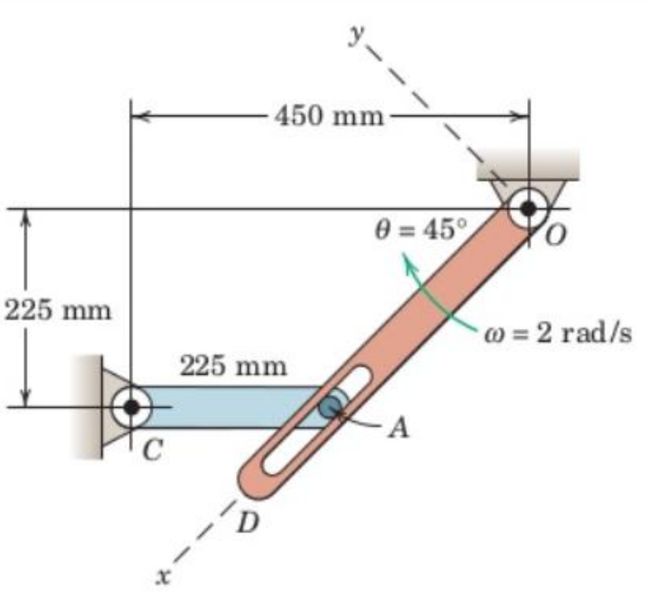
It equal $2\,\vec{w} \times \vec{v}_{rel}$.
It represent the difference between the acceleration of A as measured from non-rotating axes and from rotating axes

If P is a coincident point with A on a rigid body. We can wrtie the following relation:

$$\vec{a}_A = \vec{a}_P + 2\,\overrightarrow{w} \times \vec{v}_{rel} + \vec{a}_{rel}$$

Example:

Determine the velocity of a pin A (VA) and the velocity of A relative to the rotating slot in OD (VA/P slop) .

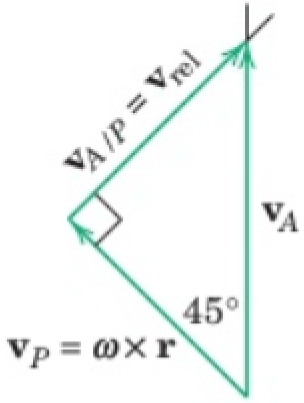


Use X-Y as rotating axis attached to OD:

Solution:

$$VA = VB + w \times r + Vres$$

Origin is fixed at O (Vz=0)



$$VA = w \times r + Vres$$

$$VA = \underset{ca}{w} \times \underset{ca}{r} = \underset{cak}{w} \times [(225)\cos 45(-i) + 225 sin45(-j)] = \underset{cak}{w} \times \frac{225}{\sqrt{2}}(-i-j) = \frac{225}{\sqrt{2}}\underset{ca}{w}(i-j)$$

$$\underset{p}{r} = \overline{OP}i = \sqrt{(45-225)^2 + 225^2}\ \ i = 450\sqrt{2}\ \ j\ \ mm/s$$

$$\underset{res}{V} = \dot{x}\ i$$

$$VA = 450\sqrt{2}\, j + \dot{x}\ i = \frac{225}{\sqrt{2}}\underset{ca}{w}\ (i-j)$$

2 equs 2 unknowns

$$\dot{x} = \frac{225}{\sqrt{2}}\underset{ca}{w} \qquad -(225/\sqrt{2})\underset{ca}{w} = 450\sqrt{2}$$

Solution:

$$\omega_{CA} = -4\, rad/s\,(ccw);$$

$$\dot{x} = v_{rel} = -450\sqrt{2}\, mm/s\,;$$

$$v_A = r\,\omega = (225)(4) = 900\, mm/s\,;$$

$$v_P = \overline{OP}\,\omega = \left(225\sqrt{2}\right)(2) = 450\sqrt{2}\, mm/s\,;$$

$$v_{A/P} = v_{rel} = 450\sqrt{2}\, mm/s\,;$$

Check :

$$v_{A/P} = v_A - v_P$$

$$450\sqrt{2} \overset{؟}{=} 900 - 450\sqrt{2}$$

$$900 \overset{؟}{=} 2\left(450\sqrt{2}\right) = 900 \;\checkmark$$

Should add vectors and not magnitudes.

CHAPTER 16

Plane Kinetics Of Rigid Bodies

⇒ Force , Mass and Acceleration

** General Equations Of Motion

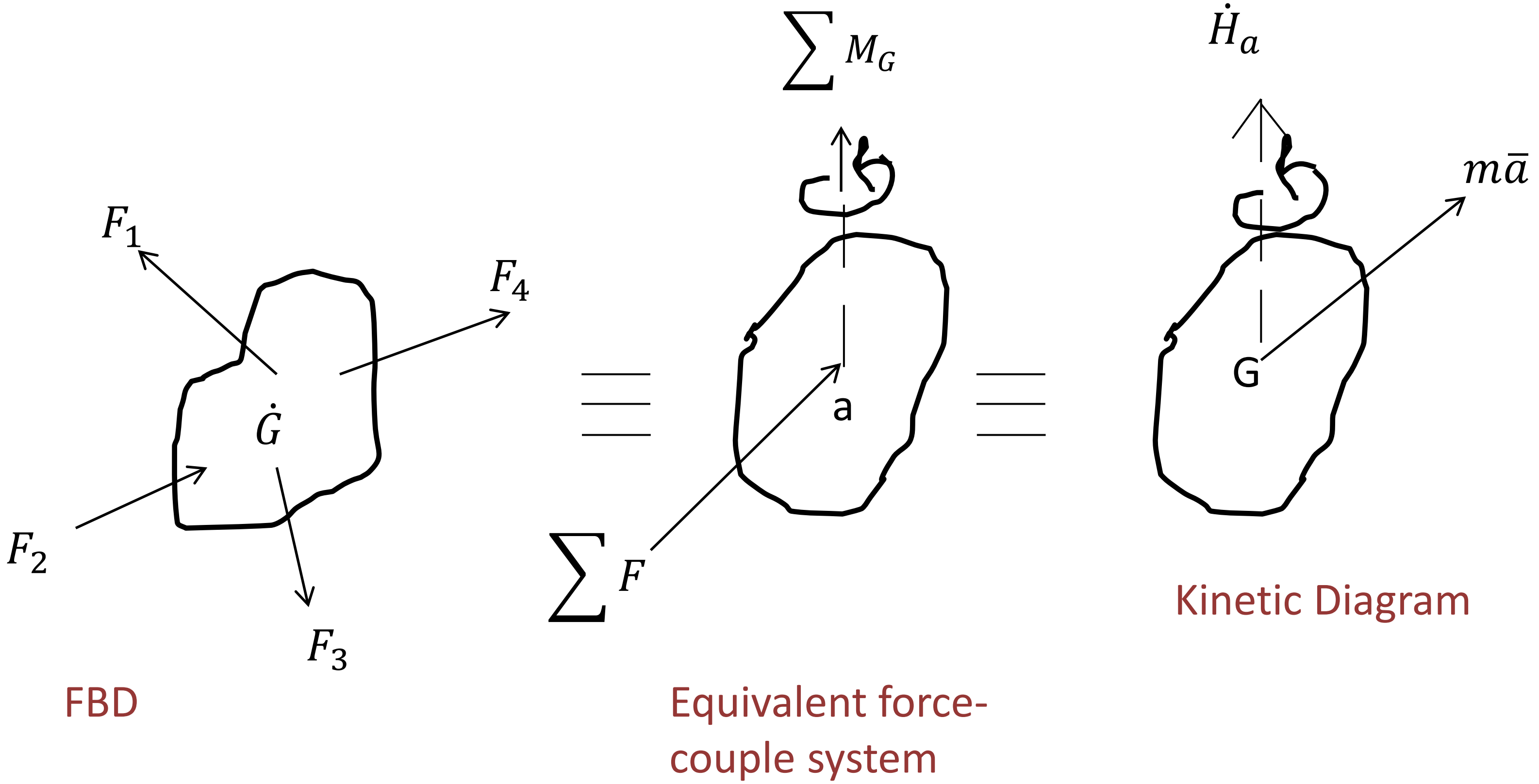
$$\Sigma\vec{F} = m\,\vec{\bar{a}}$$

Where :

m : $Mass\ of\ rigid\ body$

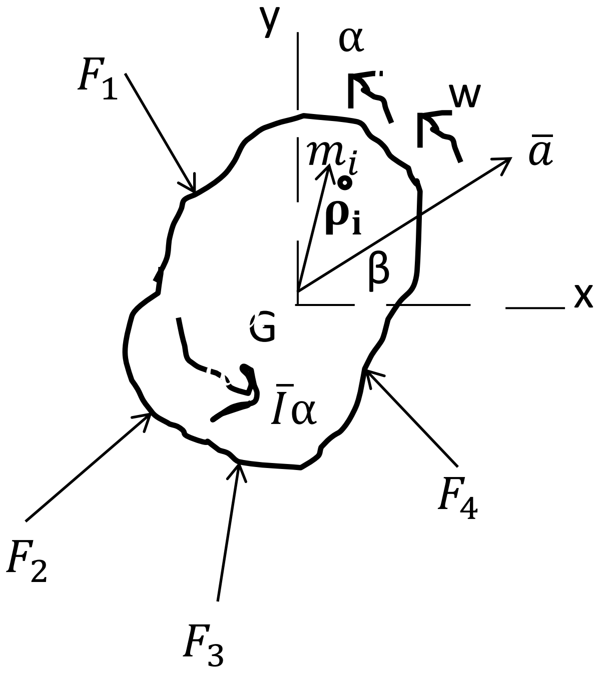
$\vec{\bar{a}}$: $Acceleration\ of\ mass\ center\ G$

$$\sum M_G = \dot{H}_c$$



F_1
F_4
$\dot{G}$
F_2
F_3
FBD
$\equiv$
$\sum M_G$
a
$\sum F$
Equivalent force-couple system
$\equiv$
$\dot{H}_a$
$m\bar{a}$
G
Kinetic Diagram

Plane-motion equations:



$$\dot{H}_G = \sum \vec{\boldsymbol{\rho}_i} \; x \; m\dot{\vec{\boldsymbol{\rho}_i}}$$

$\vec{w} \; x \; \vec{\boldsymbol{\rho}_i}$

Position vector

$$\dot{H}_G = \sum \vec{\boldsymbol{\rho}_i} \; x \; m_i(\vec{w} \; x \; \vec{\boldsymbol{\rho}_i})$$

$$|H_G| = \sum \vec{\boldsymbol{\rho}_i}^{\,2} \; x \; m_i \; x \; \vec{w}$$

$$= \sum \boldsymbol{\rho}_i^{\,2} \; x \; m_i \longrightarrow \int \boldsymbol{\rho}_i^2 \, \boldsymbol{dm}$$

Mass moment of inertia

$\bar{I}$

about z axis

➔ $H_G = \bar{I}\,w$

$(G = mv)$

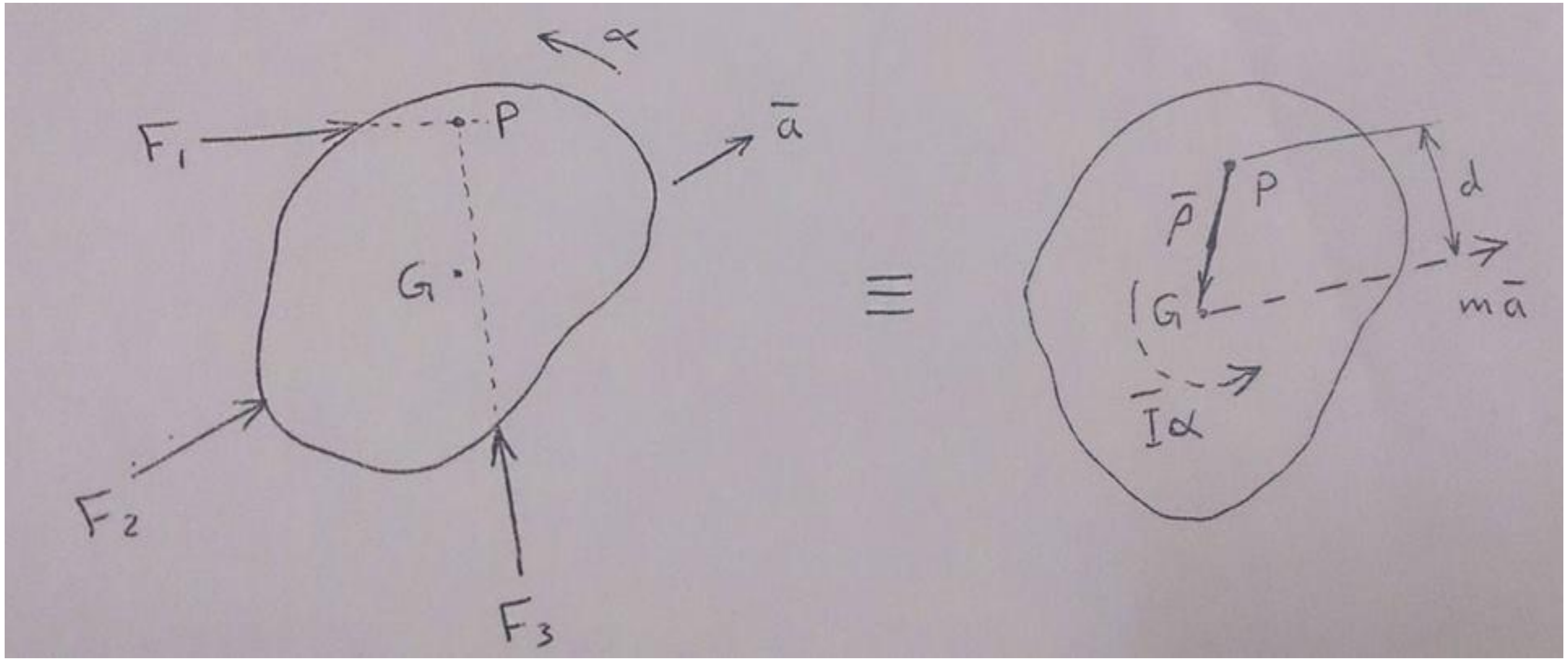
$$\sum M_G = \dot{H}_G = \bar{I}\dot{w} = \bar{I}\alpha$$

$$\left(\sum F_a = m\dot{a}\right)$$

Ch-6   plane kinetics of Rigid Bodies
Force, Mass and Acceleration
General Eqns of Motion

<u>Alternative moment equ.</u>

$$\sum \overrightarrow{Mp} = \dot{\overrightarrow{HG}} + \vec{\bar{\rho}} \times m\vec{\bar{a}}$$



$$\sum Mp = \bar{I}\alpha + m\bar{a}d$$

In terms of Ip (moment of inertia a bout p):

$$\sum \overrightarrow{Mp} = Ip\vec{\alpha} + \overrightarrow{\bar{\rho}} \times m\overrightarrow{\bar{a}p}$$

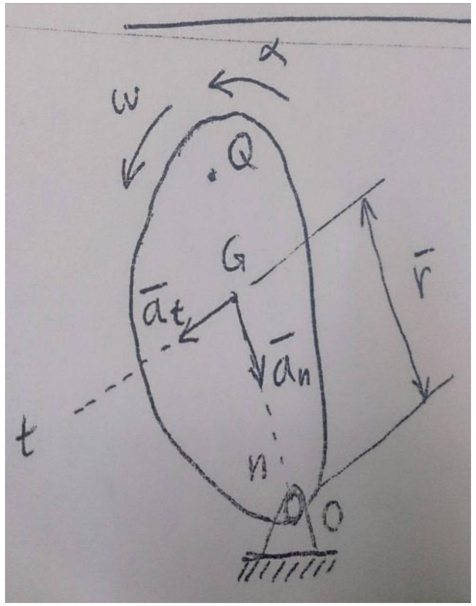
If P is a fixed point ($_o$) with = 0

$$\sum M_o = I_o \alpha$$

For a system of interconnected bodies:

$$\sum Mp = \sum \overline{I\alpha} + \sum m\bar{a}\, d$$

<u>Fixed-Axis Rotation</u>



$$\bar{a}\, t = \bar{r}\alpha$$

$$\bar{a}\, n = \bar{r}w^2$$

$$\sum MG = \bar{I}\alpha$$

$$\sum M_o = I_o\alpha$$

Q is called 'center of percussion'.

Because $\sum MQ = 0$

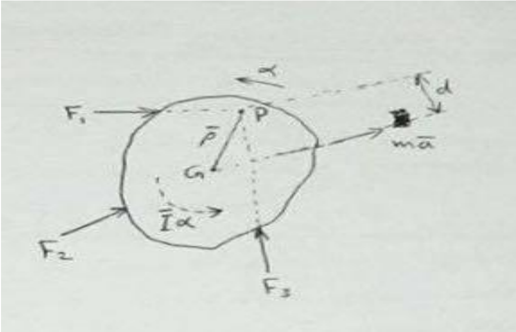
As the resultant force pass through it

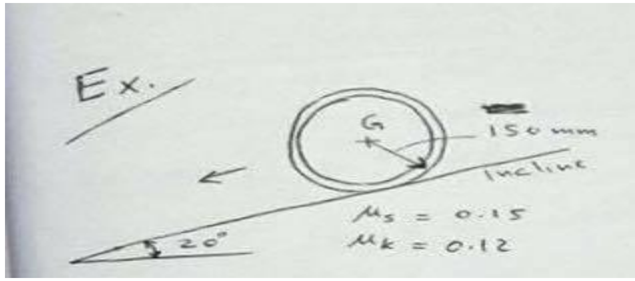
General plane motion

$\sum f = m\bar{a}$

$\sum M_G = \bar{I}\alpha$

$\sum M_P = I\alpha + m\bar{a}d$

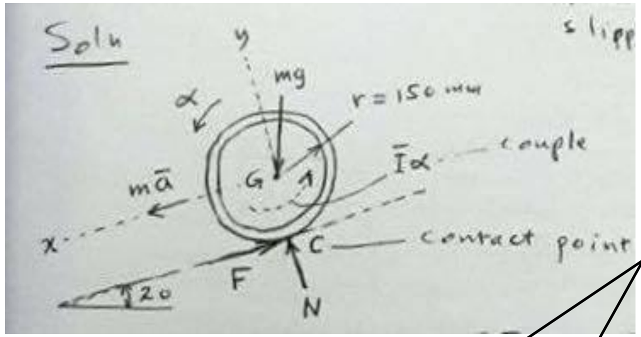




Ex.

Metal hoop is released from rest find α and time to move 3 m ,

Assume the hoop rolls without slipping.



solu

$\bar{a} = r\alpha$

$\sum f_x = m\bar{a}_x$

$mg \sin 20 - f = m\bar{a}$

$N - mg \cos 20 = 0$

$Fr = mr^2\alpha$ $\quad f = mr^{\not{2}}\alpha/\not{r} = mr\alpha$

$\sum f_y = m\bar{a}_y = 0$

$\sum M_G = \bar{I}\alpha$

$f = mg \sin 20 - m\bar{a}$ ➡ $\not{m}(g \sin 20 - \bar{a}) = \not{m}r\alpha$ ➡

$\bar{a} = g \sin 20 - r\alpha = g \sin 20 - \bar{a}$ ➡ $2\bar{a} = g \sin 20$ ➡

$\bar{a} = (g \sin 20)/2 = (9.81/2)(0.342) = 1.678\ /s^2$

$\alpha = \bar{a}/r = (1.678\ \not{m}/s^2)/(150 \times 10^{-3}\ \not{m}) = 11.2\ tad/s^2$

$x = \frac{1}{2}(a t^2)$ ➡ $t = (2x/a)^{1/2} = (2(3\not{m})/1.678\ \not{m}/s^2)^{1/2} = 1.89\ s$

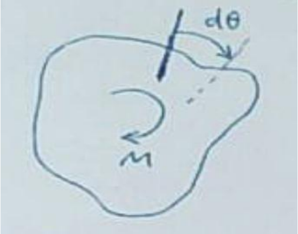
Work and Energy./Rigid bodies.

<u>Work –Energy Relations</u>

(a) <u>Work of forces and couples:</u>

$$U = \int \vec{F}.d\vec{r} = \int F\cos\alpha.ds$$

Force in the direction of displacement.



$$U = \int M.d\theta$$

Couple

(b) <u>Kinetic Energy</u> $T = \frac{1}{2}mv^2$ → **Translation**

Fixed- axis rotation: $T = \frac{1}{2}I_o w^2$

General plane Motion: $T = \frac{1}{2}m\vec{v}^2 + \frac{1}{2}Iw^2$

Angular velocity

Velocity of the Center of mass G

Moment of inertia about Mass center G

Also $T = \frac{1}{2} I_c w^2$

Moment of inertia about C (Instantaneous center of zero velocity).

(c) Potential Energy

work-energy equation: $U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e$

Work — **Kinetic** — **Potential energy Due to gravity** — **Elastic potential energy usually zero for rigid body.**

(d) Power:

$$P = \vec{F}.\vec{V} = \frac{dV}{dt}$$

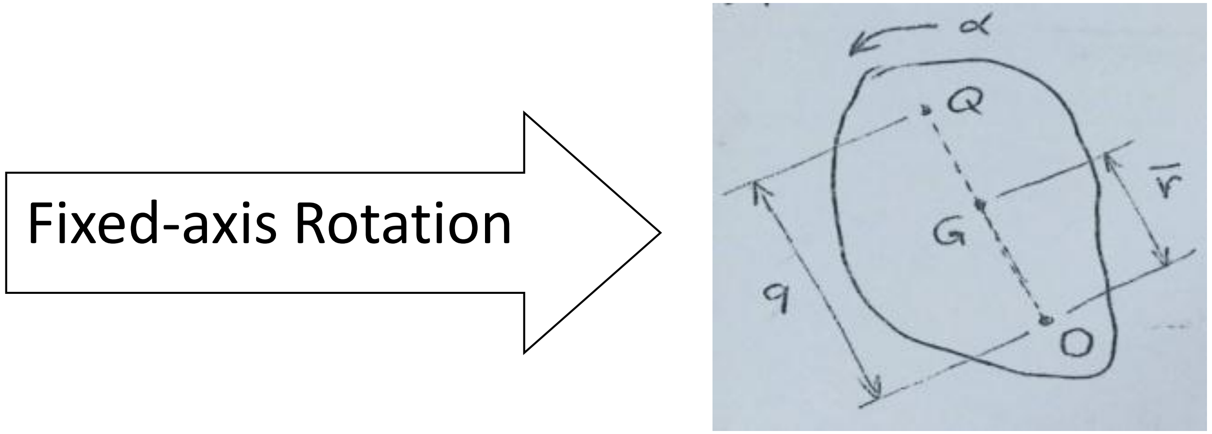
- $Power = \frac{\partial v}{\partial t} = \frac{M\partial\theta}{\partial t} = \overrightarrow{M\omega}$, due to a couple M.
- $Total\ power = F.V + M\omega,$
- **Virtual Work**: It is the work calculated using a virtual displacement and (assumed), linear or angular.

- **Center of percussion** (Q): The resultant of all forces applied to the body must pass through it. The sum of the moments of all forces about the center of percussion is always ZERO.

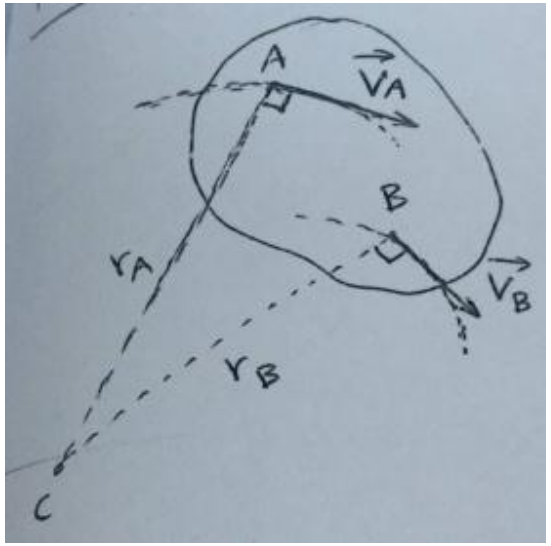
- $I = \int r^2 \partial m \, K = \sqrt{\frac{I}{m}} = Radius\ of\ gyaration$

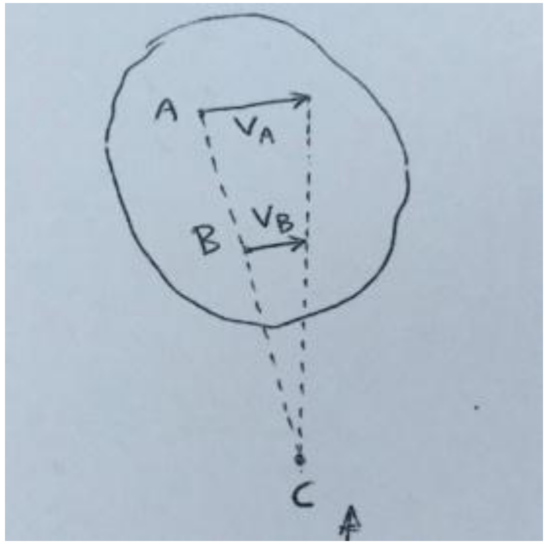
- $I. = K.^2\, m \sum M. = I. \propto$

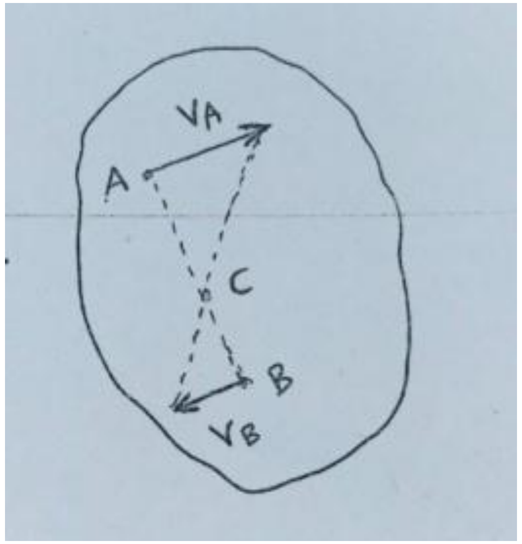
- $q = \frac{K.^2}{r}$



- What is I (moment of inertia?) resistance to rotation → H.=I.w
- Instantaneous center of zero velocity (not acceleration)!







Parallel velocities

End of the course

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