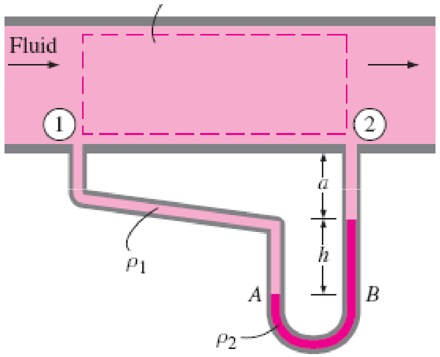
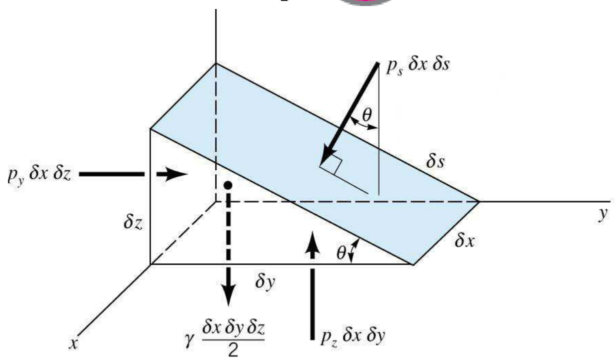
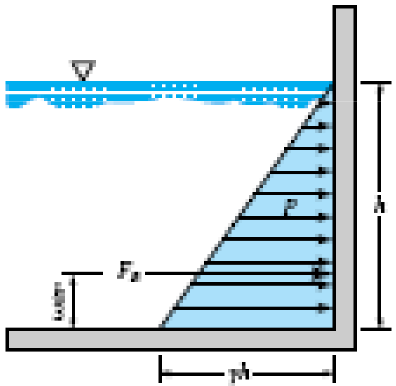
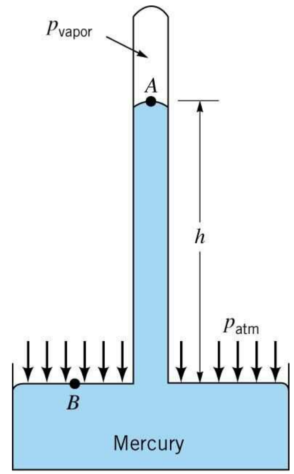
Fluid Mechanics

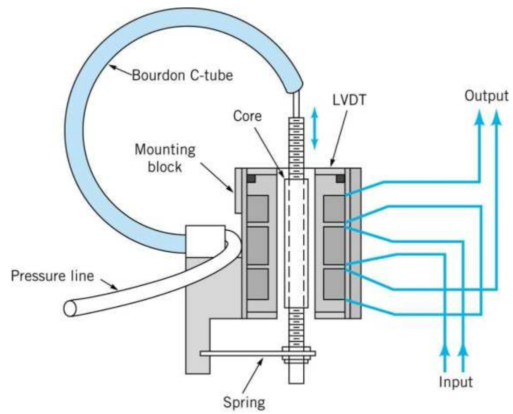
Pressure and Fluid Static











Dr. Yousef Mubarak

Content

Introduction

- By definition, fluid static means that the fluid is at rest.
- Or, or moving in such a manner that there is no relative motion between adjacent particles.
- No shearing forces is placed on the fluid.
- There are only pressure forces that act perpendicular to the surface, and no shear.
- Results in relatively “simple” analysis
- Generally look for the pressure variation in the fluid

The target is to investigate pressure and its variation throughout a fluid and the effect of pressure on submerged surfaces

Pressure

- All fluids are composed of energetic molecules in motion.
- When these molecules collide with a surface, they exert a normal and tangential force on the surface due to the change in momentum of colliding molecules
- **Pressure** is defined as a normal force exerted by a fluid per unit area (N/m^2)

$$1\ Pa = 1\ N/m^2 \qquad 1\ kPa = 10^3\ Pa$$

$$1\ bar = 10^5\ Pa = 0.1\ MPa = 100\ kPa = 14.696\ psi$$

$$1\ atm = 101{,}325\ Pa = 101.325\ kPa = 1.01325\ bars$$

$$1\ atm = 760\ torr = 760\ mmHg$$

- Pressure is also used for solids as synonymous to normal stress, which is force acting perpendicular to the surface per unit area

Pressure

Absolute, gage, and vacuum pressures

- Actual pressure at a give point is called the **absolute pressure**.
- It is measured relative to absolute vacuum (absolute zero pressure)
- Most pressure-measuring devices are calibrated to read zero in the atmosphere, and therefore indicate **gage pressure**,

$$P_{gage}=P_{abs} - P_{atm}$$

- Pressure below atmospheric pressure are called **vacuum pressure**,

$$P_{vac}=P_{atm} - P_{abs}$$

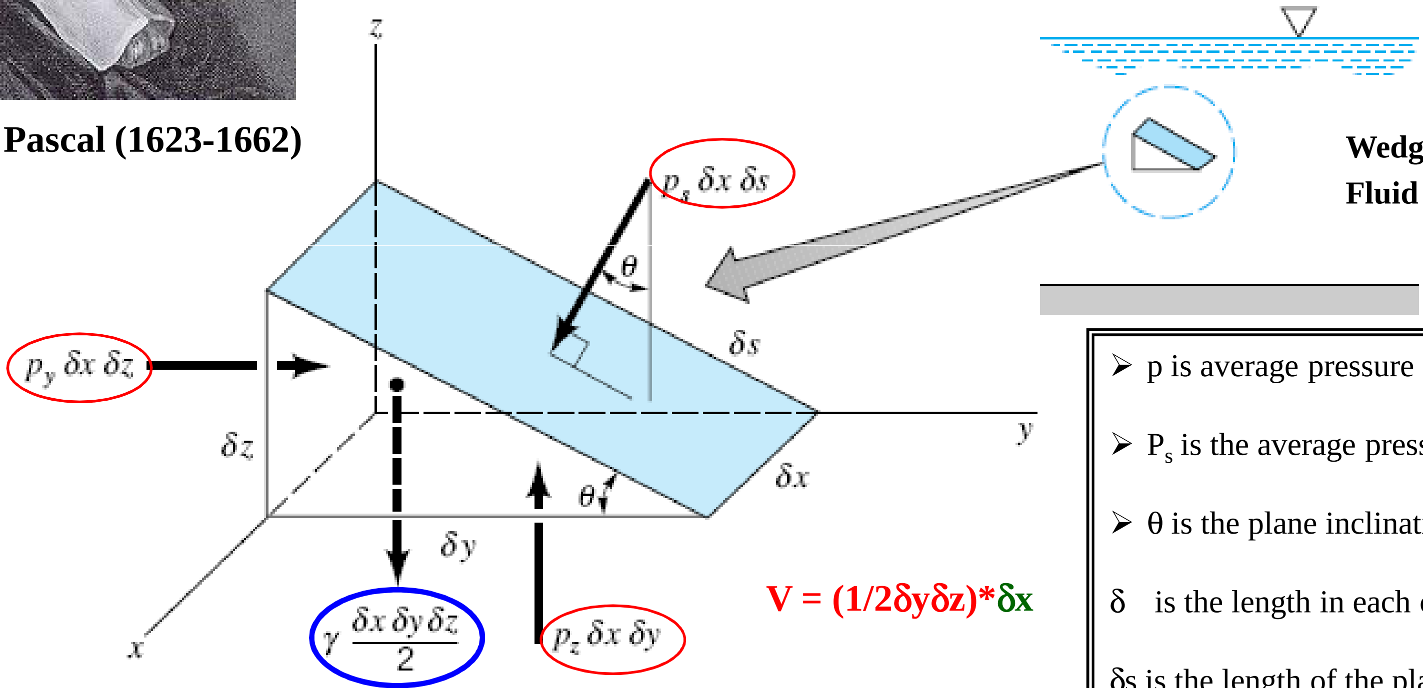
Pressure at a Point

- Pressure at any point in a fluid is the same in all directions.
- Pressure has a magnitude, but not a specific direction, and thus it is a scalar quantity

Pressure at a Point: Pascal's Law

Blaise Pascal (1623-1662)

- How does the pressure at a point vary with orientation of the plane passing through the point?



Wedged Shaped Fluid Mass

$V = (1/2\delta y\delta z)*\delta x$

- p is average pressure in the x, y, and z direction.
- P_s is the average pressure on the surface
- θ is the plane inclination

δ   is the length in each coordinate direction, x, y, z

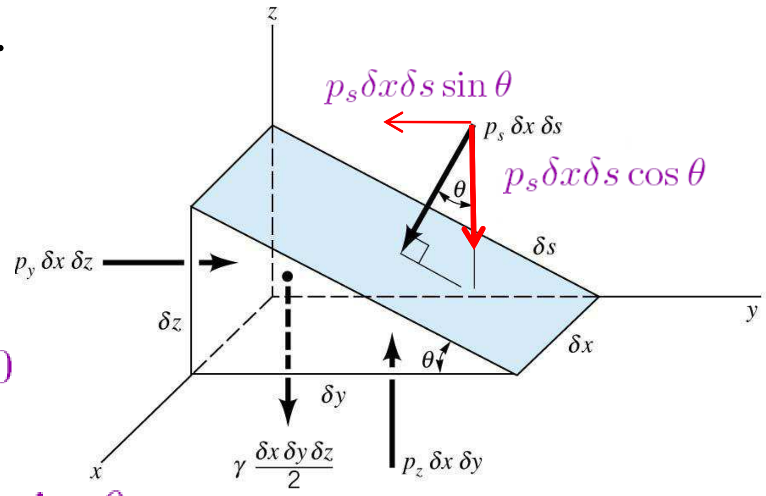
δs is the length of the plane

γ is the specific weight

- The only external forces acting on the wedge are due to the pressure and the weight (no shearing forces)

Pressure at a Point: Pascal's Law

- The forces acting on the wedge are in equilibrium.



From Newton's second law $\sum F = ma$

$$\sum F_y = p_y \delta x \delta z - p_s \delta x \delta s \sin\theta = 0$$

$$\sum F_z = p_z \delta x \delta y - p_s \delta x \delta s \cos\theta - \gamma \frac{\delta x \delta y \delta z}{2} = 0$$

- From geometry $\delta y = \delta s \cos\theta$ $\delta z = \delta s \sin\theta$

$$p_y \delta x \delta z = p_s \delta x \delta z \longrightarrow p_y = p_s$$

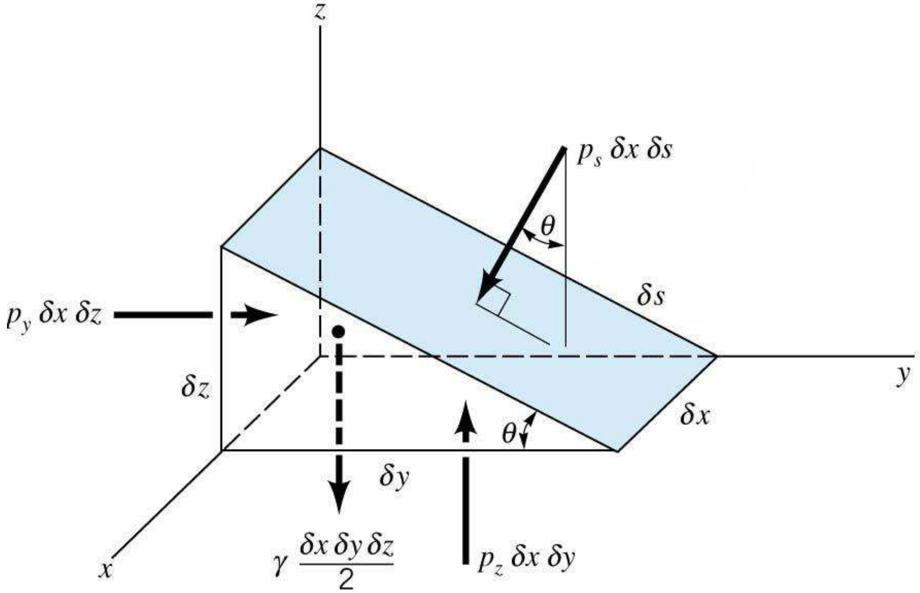
$$p_z \delta x \delta y - p_s \delta x \delta y = \gamma \frac{\delta x \delta y \delta z}{2} \longrightarrow p_z - p_s = \gamma \frac{\delta z}{2}$$

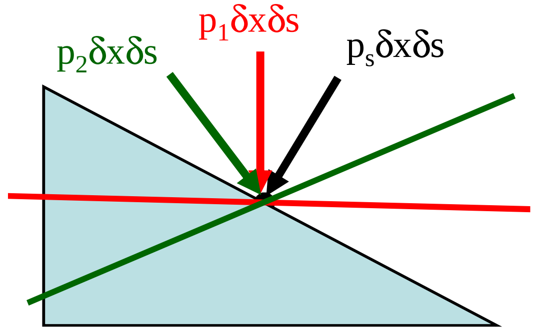
- when the size of the wedge goes to zero $\Delta z \rightarrow 0$

$$p_z = p_s$$

Pressure at a Point: Pascal's Law

➢ Pascal's Law: the pressure at a point in a fluid at rest, or in motion, is independent of the direction as long as there are no shearing stresses present.





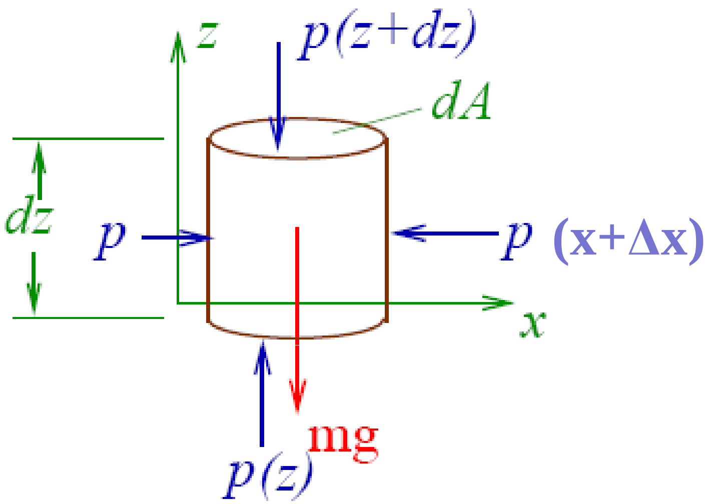
$$p_s = p_1 = p_2$$

Pressure Field Equations

- How does the pressure in a fluid in which there are no shearing stresses vary from point to point?

$$\sum \delta \mathbf{F} = \delta m\, \mathbf{a}$$

For a fluid at rest a = 0.0



$$mg - p(z)dA + p(z+dz)dA = 0$$

$$(\rho dz dA)g - p(z)dA + p(z+dz)dA = 0$$

$$(\rho dz)g - p(z) + p(z+dz) = 0$$

$$\gamma = \rho g$$

$$\gamma dz - p(z) + p(z+dz) = 0$$

$$\frac{dP}{dz} = \lim_{\Delta z \to 0} \frac{(P)_z - (P)_{z+\Delta z}}{\Delta z} = -\rho g = -\gamma$$

$$\boxed{\frac{dp}{dz} = -\gamma}$$

the Hydrostatic Equation

In the same manner

$$\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial p}{\partial y} = 0$$

Hydrostatic Condition: Physical Implications

- Pressure changes with elevation
- Pressure does not change in the horizontal x-y plane
- The pressure gradient in the vertical direction is negative
- The pressure decreases as we move upward in a fluid at rest
- Pressure in a liquid does not change due to the shape of the container
- Specific Weight γ does not have to be constant in a fluid at rest
- Air and other gases will likely have a varying γ
- Thus, fluids could be incompressible or compressible statically

Hydrostatic Condition: Incompressible Fluids

- The specific weight changes either through ρ, density or *g*, gravity.
- The change in g is negligible, and for liquids ρ does not vary appreciable, thus most liquids will be considered incompressible.

Starting with the Hydrostatic Equation: $\frac{dp}{dz} = -\gamma$

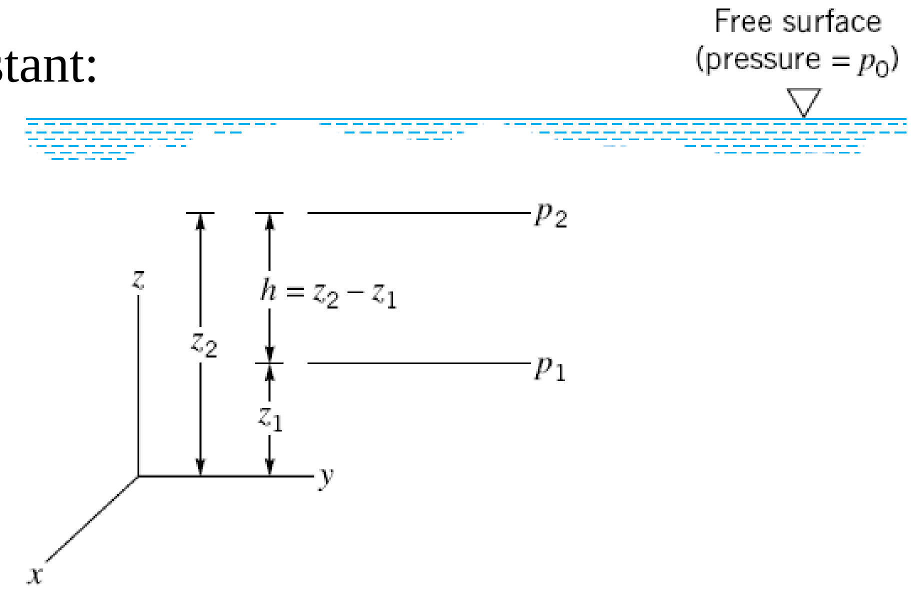
We can immediately integrate since γ is a constant:

$$\int_{p_1}^{p_2} dp = -\gamma \int_{z_1}^{z_2} dz$$

$$p_2 - p_1 = -\gamma(z_2 - z_1)$$

$$p_1 - p_2 = \gamma(z_2 - z_1)$$

$$\Delta P = P_2 - P_1 = \rho g \, \Delta z = \gamma_s \, \Delta z$$



- Where the subscripts 1 and 2 refer two different vertical levels as in the schematic.

Hydrostatic Condition: Incompressible Fluids

- As in the schematic, noting the definition of h = $z_2 - z_1$:

$$p_1 - p_2 = \gamma h$$

Linear Variation with Depth

$$p_1 = \gamma h + p_2$$

- h is known as the pressure head. The type of pressure distribution is known as a hydrostatic distribution. The pressure must increase with depth to hold up the fluid above it, and *h* is the depth measured from the location of p_2.

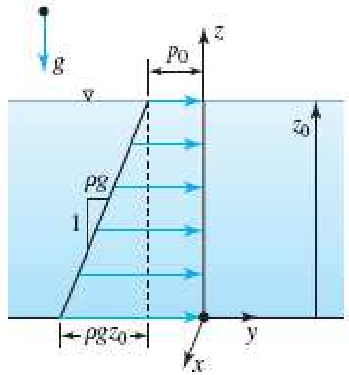
- The equation for the pressure head is the following:

$$h = \frac{p_1 - p_2}{\gamma}$$

- Physically, it is the height of the column of fluid of a specific weight, needed to give the pressure difference $p_1 - p_2$.

Hydrostatic Condition: Incompressible Fluids

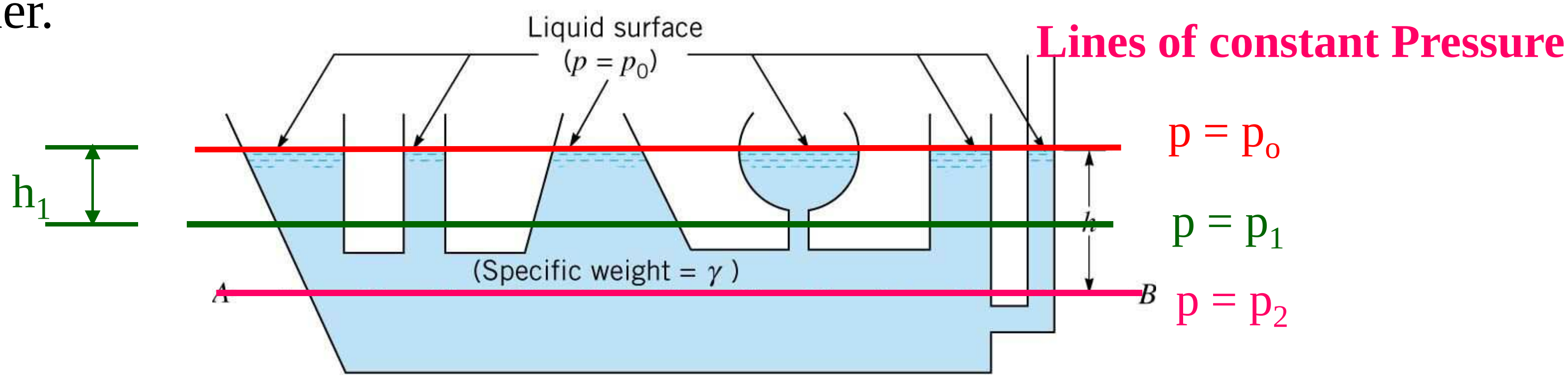
- If we are working exclusively with a liquid, then there is a free surface at the liquid-gas interface. For most applications, the pressure exerted at the surface is atmospheric pressure, p_o. Then the equation is written as follows:



$$p = \gamma h + p_0$$

For incompressible, the pressure will vary linearly with depth

- The Pressure in a homogenous, incompressible fluid at rest depends on the depth of the fluid relative to some reference and is not influenced by the shape of the container.



For $p_2 = p = \gamma h + p_o$

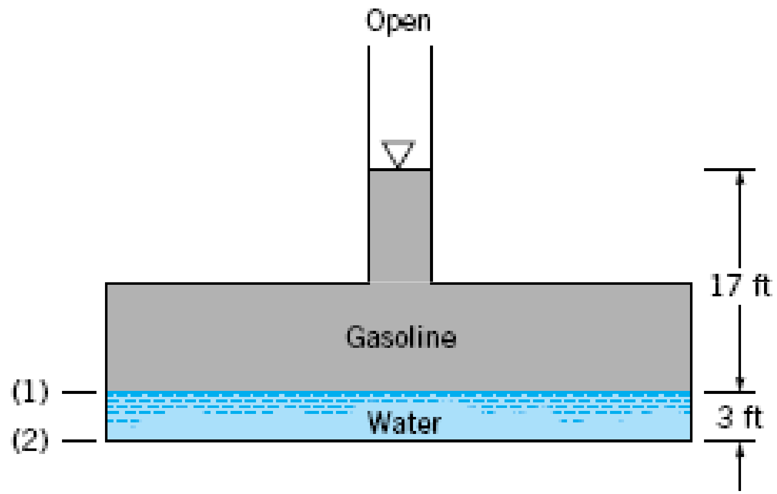
For $p_1 = p = \gamma h_1 + p_o$

Example

- Because of a leak in a buried gasoline storage tank, water has seeped in to the depth shown. If the specific gravity of the gasoline is $SG = 0.68$,. determine the pressure at the gasoline-water interface and at the bottom of the tank.

$$p = \gamma h + p_0$$ liquids at rest

$$p_1 = SG\gamma_{H_2O} h + p_0$$
$$= (0.68)(62.4 \text{ lb/ft}^3)(17 \text{ ft}) + p_0$$
$$= 721 + p_0 \text{ (lb/ft}^2\text{)}$$



The gage pressure $p_1 = 721 \text{ lb/ft}^2$

$$\frac{p_1}{\gamma_{H_2O}} = \frac{721 \text{ lb/ft}^2}{62.4 \text{ lb/ft}^3} = 11.6 \text{ ft}$$

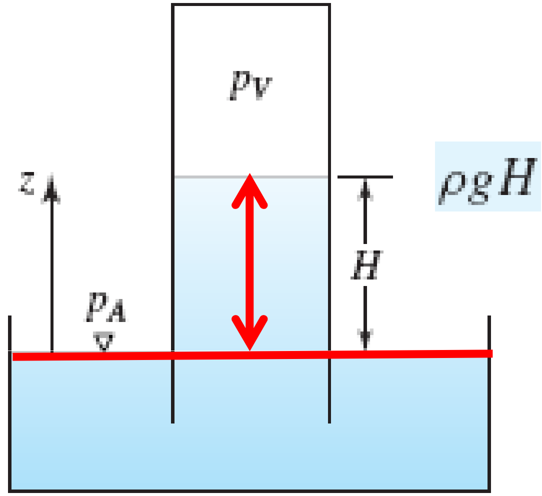
$$p_2 = \gamma_{H_2O} h_{H_2O} + p_1$$
$$= (62.4 \text{ lb/ft}^3)(3 \text{ ft}) + 721 \text{ lb/ft}^2$$
$$= 908 \text{ lb/ft}^2$$

➡

$$\frac{p_2}{\gamma_{H_2O}} = \frac{908 \text{ lb/ft}^2}{62.4 \text{ lb/ft}^3} = 14.6 \text{ ft}$$

Example

- An evacuated tube is immersed in a liquid as shown. How high will the liquid rise into the tube? Calculate this height for water and mercury. You can neglect surface tension in the solution of this problem.



$$\boldsymbol{P_A = \rho g H + P_V}$$

The vapor pressures for water and mercury (at 20°C) are 2.34×10^{-3} and $1.1\times10^{-3}\ N/m^2$

For water:

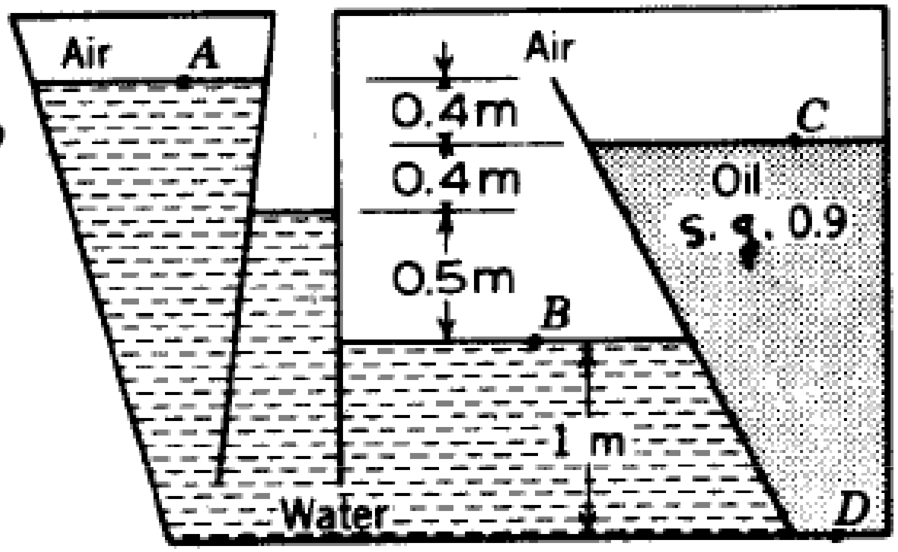
$$H = \frac{p_A - p_v}{\rho g} = \frac{1.01\times10^5\ \text{N/m}^2 - 2.34\times10^3\ \text{N/m}^2}{(998\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} = 10.1\ \text{m}$$

For mercury (Hg):

$$H = \frac{p_A - p_v}{\rho g} = \frac{1.01\times10^5\ \text{N/m}^2 - 1.1\times10^{-3}\ \text{N/m}^2}{(13{,}550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} = 0.76\ \text{m}$$

Example

Calculate the pressure, in kPa, at A, B, C, and D



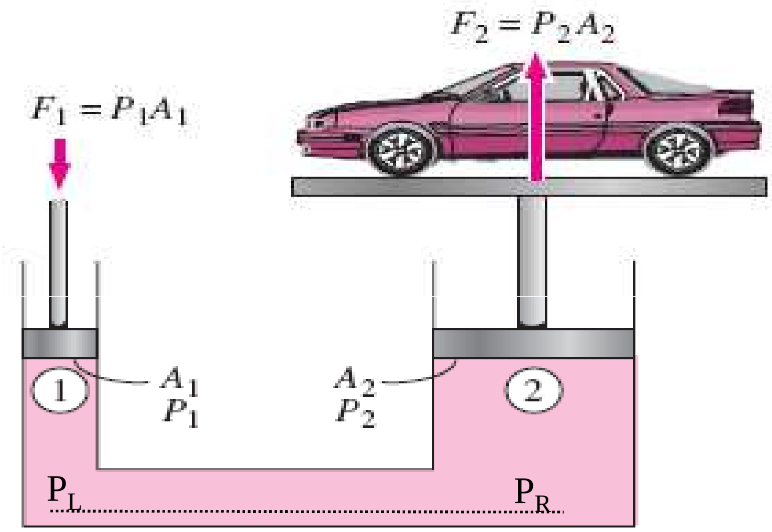
$p_A = -(0.4 + 0.4)(9.790) = -7.832$ kPa; $p_B = (0.5)(9.790) = 4.895$ kPa.

Neglecting air, $p_C = p_B = 4.895$ kPa;

$p_D = 4.895 + (0.9)(9.790)(1 + 0.5 + 0.4) = 21.636$ kPa.

Hydrostatic Application: Transmission of Fluid Pressure

- Mechanical advantage can be gained with equality of pressures
- A small force applied at the small piston is used to develop a large force at the large piston.
- This is the principle between hydraulic jacks, lifts, presses, and hydraulic controls
- Mechanical force is applied through jacks action or compressed air for example



$$F_2 = \frac{A_2}{A_1} F_1$$

$$p_l = p_1 + \rho g z$$

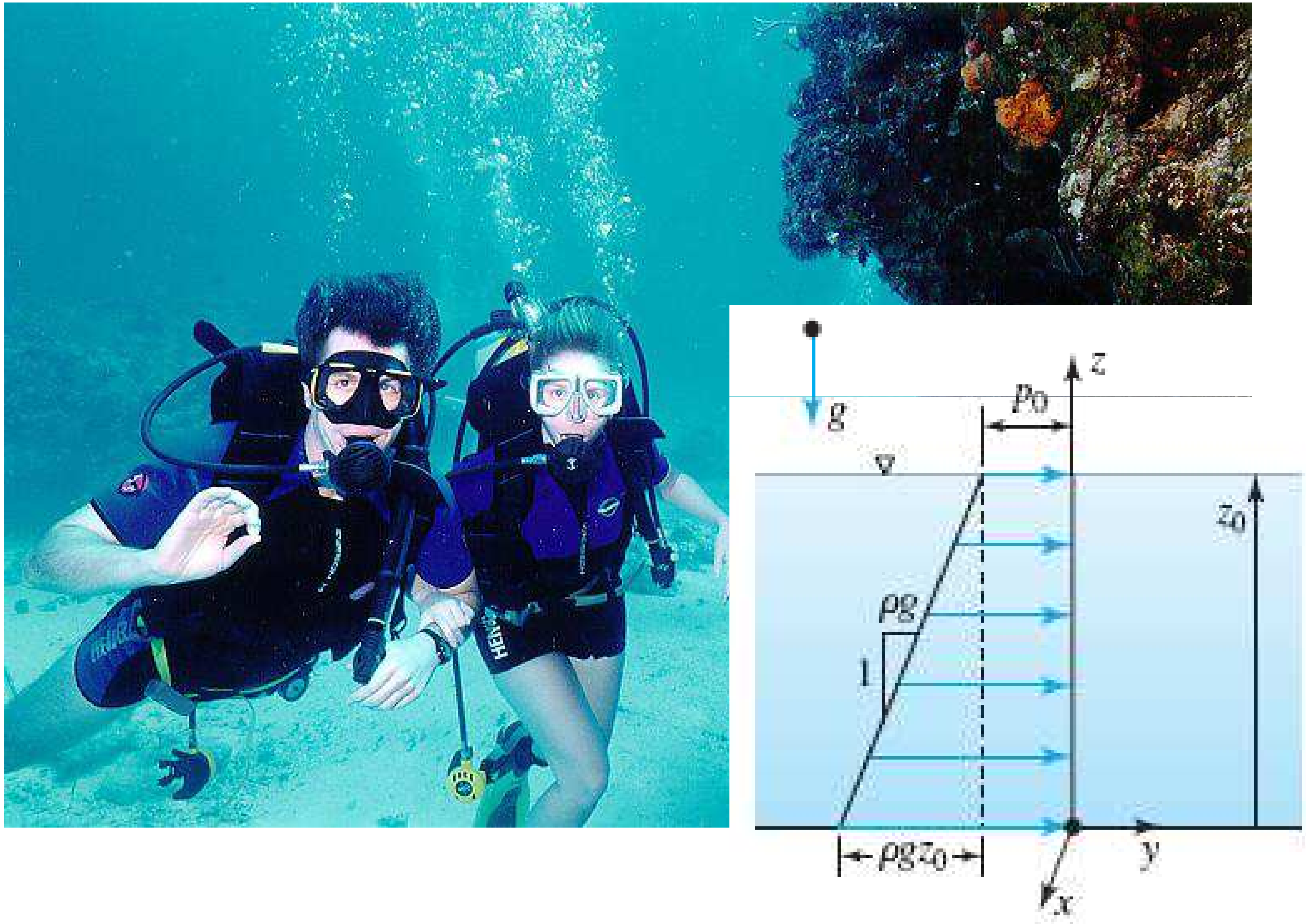
$$p_r = p_2 + \rho g z$$

$$p_1 + \rho g z = p_2 + \rho g z$$

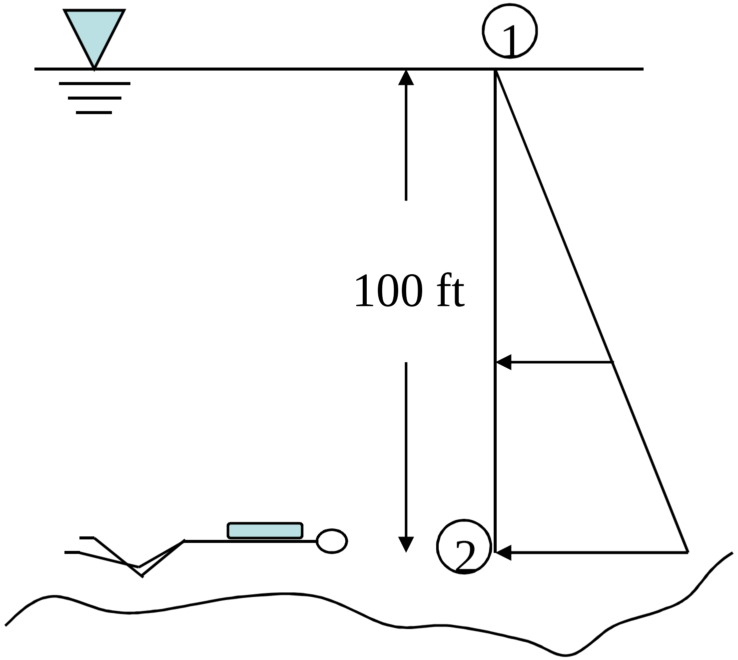
$$p_1 = p_2$$

- Ratio A_2/A_1 is called ideal mechanical advantage

Scuba Diving and Hydrostatic Pressure



Scuba Diving and Hydrostatic Pressure



Pressure on diver at 100 ft?

$$P_{gage,2} = \rho g z = \left(998 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)(100\,ft)\left(\frac{1m}{3.28\,ft}\right)$$

$$= 298.5 kPa \left(\frac{1atm}{101.325 kPa}\right) = 2.95 atm$$

$$P_{abs,2} = P_{gage,2} + P_{atm} = 2.95 atm + 1 atm = 3.95 atm$$

Danger of emergency ascent?

$$P_1V_1 = P_2V_2 \qquad \text{Boyle's law}$$

$$\frac{V_1}{V_2} = \frac{P_2}{P_1} = \frac{3.95 atm}{1 atm} \approx 4$$

- If you hold your breath on ascent, your lung volume would increase by a factor of 4, which would result in embolism and/or death.

Hydrostatic Condition: Compressible Fluids

- Gases such as air, oxygen and nitrogen are thought of as compressible, so we must consider the variation of density in the hydrostatic equation:

$$\frac{dp}{dz} = -\gamma$$

Note: $\gamma = \rho g$ and not a constant, then $\frac{dp}{dz} = -\rho g$

By the Ideal gas law: $p = \rho RT$ Thus, $\rho = \frac{p}{RT}$ $\rho = \frac{p * Mwt}{RT}$

R is the Gas Constant

T is the temperature

ρ is the density (mole /m^3)

ρ is the density (Kg /m^3)

Then, $\frac{dp}{dz} = -\frac{gp}{RT}$

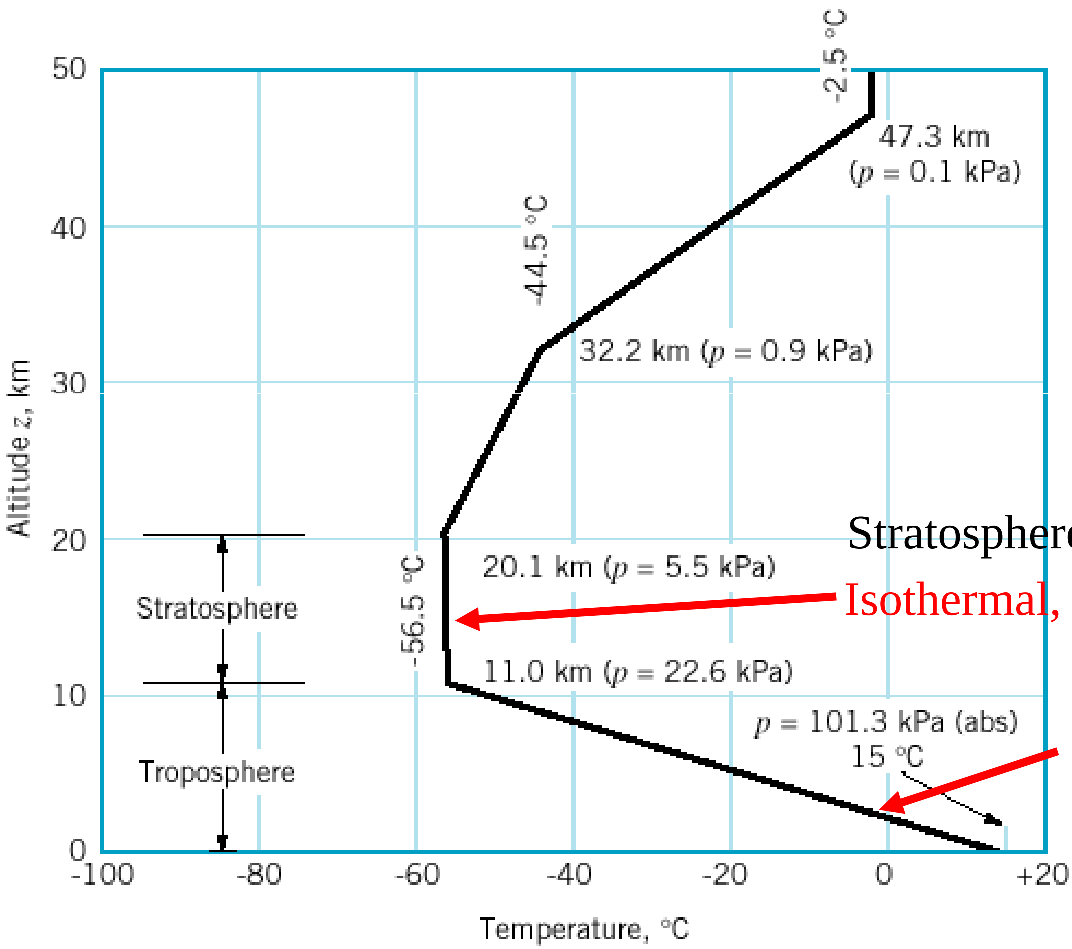
$$\int_{p_1}^{p_2} \frac{dp}{p} = \ln \frac{p_2}{p_1} = -\frac{g}{R}\int_{z_1}^{z_2} \frac{dz}{T}$$

For Isothermal Conditions, T is constant, T_o:

$$\left[\ln(p)\right]_{p_1}^{p_2} = -\left[\frac{gz}{R^*T}\right]_{z_1}^{z_2} \longrightarrow p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$$

Hydrostatic Condition: U.S. Standard Atmosphere

➢ Idealized Representation of the Mid-Latitude Atmosphere



Standard Atmosphere is used in the design of aircraft, missiles and spacecraft.

Stratosphere:

Isothermal, $T = T_o$

Troposphere:

Linear Variation, $T = T_a - \beta z$

Hydrostatic Condition: U.S. Standard Atmosphere

Starting from, $$\int_{p_1}^{p_2} \frac{dp}{p} = \ln \frac{p_2}{p_1} = -\frac{g}{R} \int_{z_1}^{z_2} \frac{dz}{T}$$

Now, for the Troposphere, Temperature is not constant:

$$T = T_a - \beta z$$

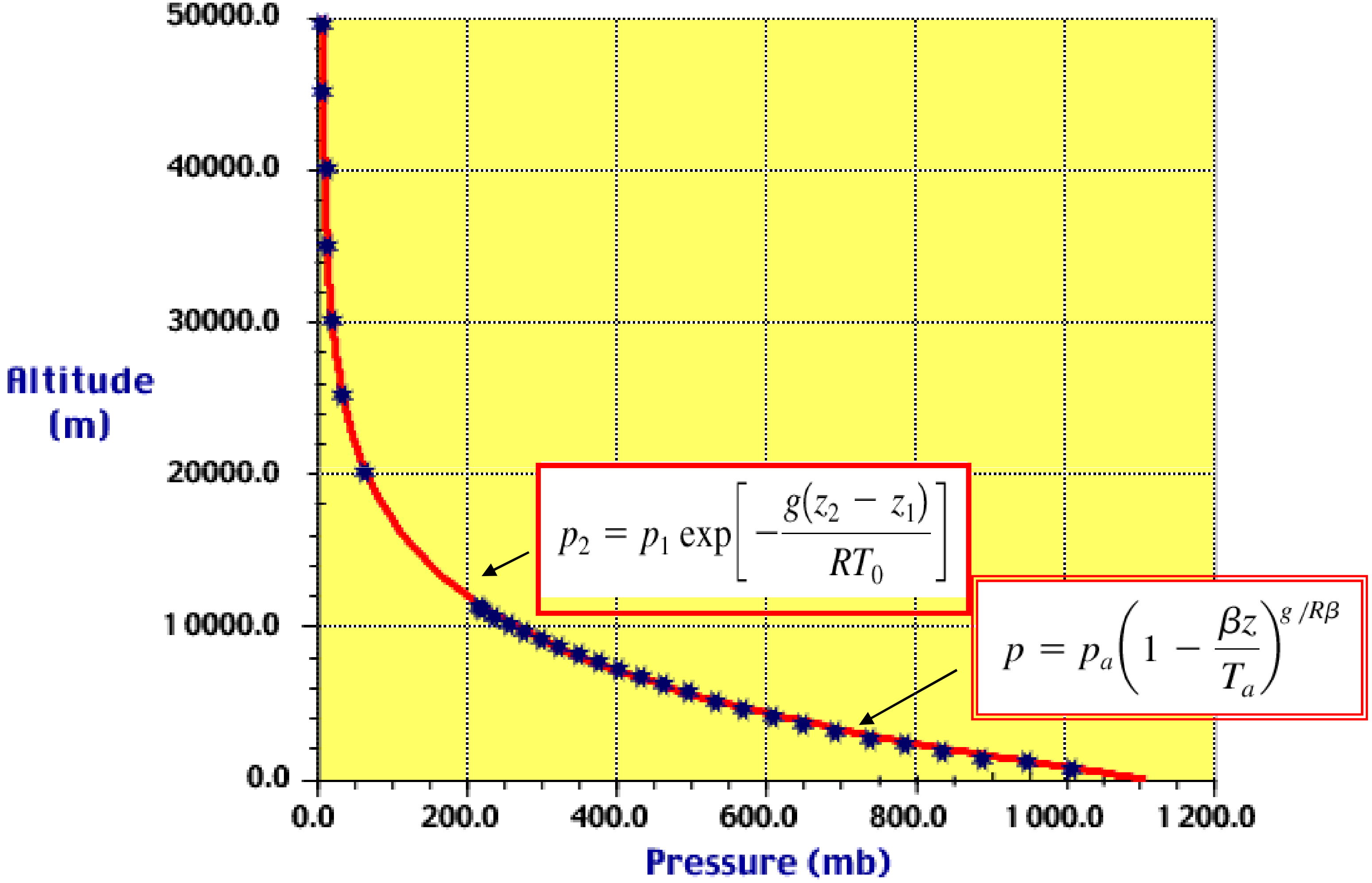
β is known as the lapse rate, 0.00650 K/m, and T_a is the temperature at sea level, 288.15 K.

Substitute for temperature and Integrate:

$$p = p_a \left(1 - \frac{\beta z}{T_a}\right)^{g/R\beta}$$

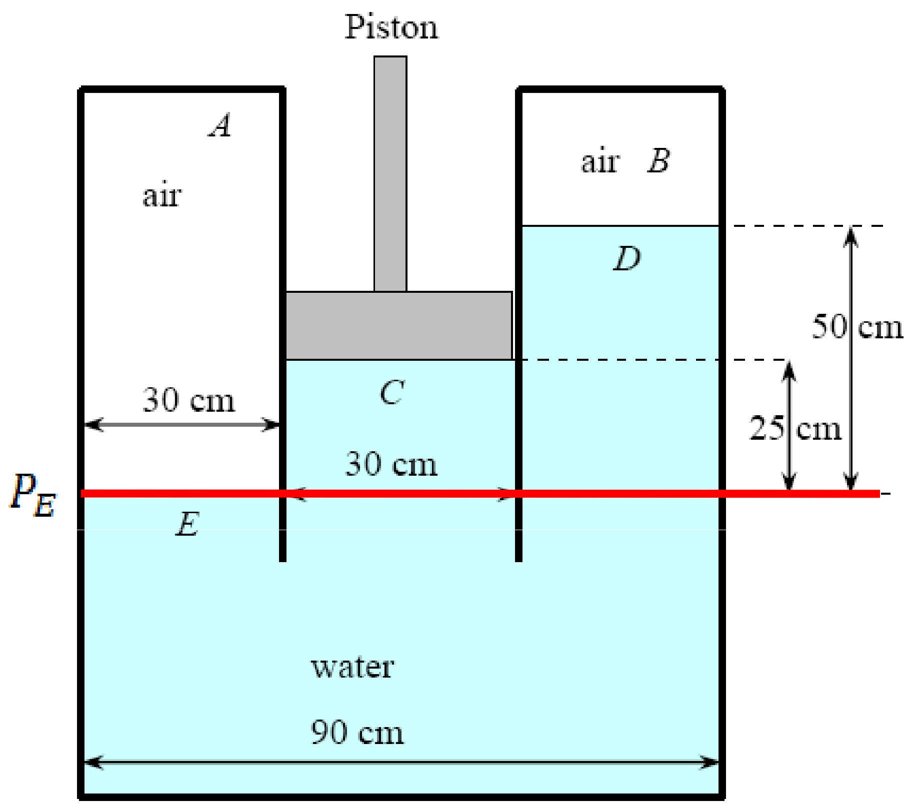
p_a is the pressure at sea level, 101.33 kPa, R is the gas constant, 286.9 J/kg.K

Pressure Distribution in the Atmosphere



Example

➢ Two chambers with the same fluid at their base are separated by a piston whose weight is 25 N, as shown. Calculate the gage pressures in chambers *A and B.*



$$P_A = P_E$$

$$P_E = 1000 * 9.81(0.5) + P_B$$

$$P_E = \rho g(0.25) + \frac{W}{A_{piston}}$$

$$P_E = 1000 * 9.81(0.25) + \frac{25}{\pi(0.3)^2/4} = 2806\ Pa$$

$$P_B = 2806 - 1000 * 9.81(0.5) = -2099\ Pa$$

Example

- The Empire State Building in New York City, one of the tallest buildings in the world, rises to a height of approximately 1250 ft. Estimate the ratio of the pressure at the top of the building to the pressure at its base, assuming the air to be at a common temperature of 59 F. Compare this result with that obtained by assuming the air to be incompressible with

  $\gamma = 0.0765 \text{ lb/ft}^3$ at 14.7 psi(abs) (values for air at standard conditions).

- For the assumed isothermal conditions, and treating air as a compressible fluid

$$\frac{p_2}{p_1} = \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$$

$$= \exp\left\{-\frac{(32.2 \text{ ft/s}^2)(1250 \text{ ft})}{(1716 \text{ ft}\cdot\text{lb/slug}\cdot{}^\circ\text{R})[(59 + 460)^\circ\text{R}]}\right\} = 0.956$$

- If the air is treated as an incompressible fluid

$$p_2 = p_1 - \gamma(z_2 - z_1)$$

$$\frac{p_2}{p_1} = 1 - \frac{\gamma(z_2 - z_1)}{p_1} = 1 - \frac{(0.0765 \text{ lb/ft}^3)(1250 \text{ ft})}{(14.7 \text{ lb/in.}^2)(144 \text{ in.}^2/\text{ft}^2)} = 0.955$$

Measurement of Pressure

- Absolute Pressure: Pressure measured relative to a perfect vacuum
- Gage Pressure: Pressure measured relative to local atmospheric pressure

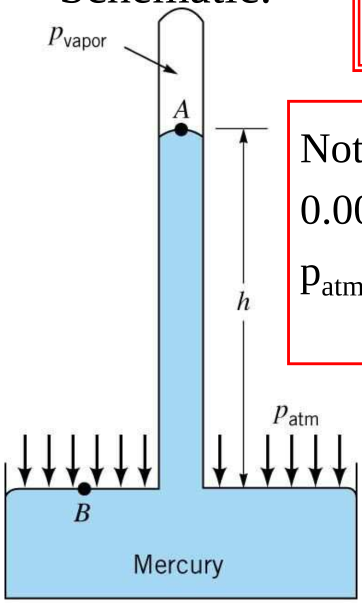
- A gage pressure of zero corresponds to a pressure that is at local atmospheric pressure.
- Absolute pressure is always positive
- Gage pressure can be either negative or positive
- Negative gage pressure is known as a vacuum or suction
- Standard units of Pressure are psi, psia, kPa, kPa (absolute)
- Pressure could also be measured in terms of the height of a fluid in a column
- Units in terms of fluid column height are mm Hg, inches of Hg, inches of H_20,etc

Measurement of Pressure: Barometers

Evangelista Torricelli (1608-1647)

➢The first mercury barometer was constructed in 1643-1644 by Torricelli. He showed that the height of mercury in a column was 1/14 that of a water barometer, due to the fact that mercury is 14 times more dense that water. He also noticed that level of mercury varied from day to day due to weather changes, and that at the top of the column there is a vacuum.

Schematic:



$$p_{atm} = \gamma h + p_{vapor}$$

Note, often p_{vapor} is very small, 0.0000231 psia at 68° F, and p_{atm} is 14.7 psi, thus:

$$p_{atm} \approx \gamma h$$

➢ The atmospheric pressure at a location is the weight of the air above that location per unit surface area. Therefore, it changes not only with elevation but also with weather conditions.

Measurement of Pressure: Manometry

- Manometry is a standard technique for measuring pressure using liquid columns in vertical or include tubes. The devices used in this manner are known as manometers.

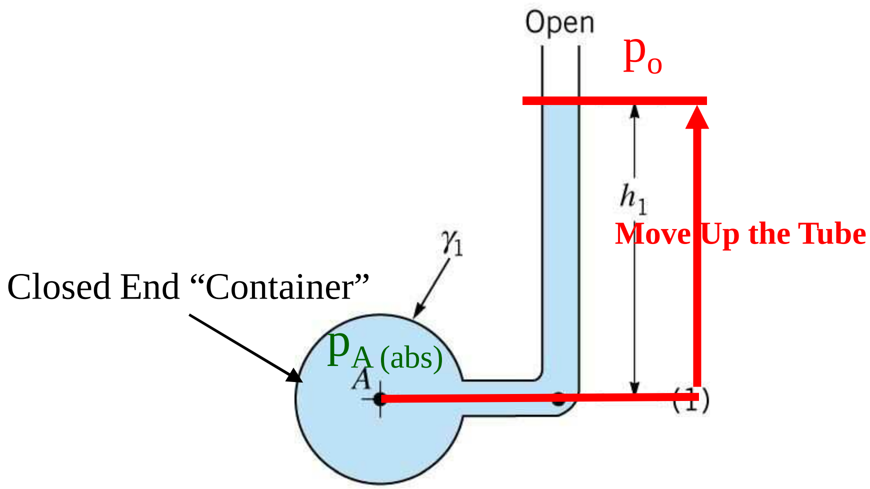
- Types of manometers:
  1) The Piezometer Tube
  2) The U-Tube Manometer
  3) The Inclined Tube Manometer

- The fundamental equation for manometers since they involve columns of fluid at rest is the following:

$$p = \gamma h + p_0$$

- *h* is positive moving downward, and negative moving upward, that is pressure in columns of fluid decrease with gains in height, and increase with gain in depth.

Measurement of Pressure: Piezometer Tube



Disadvantages:

1. **The pressure in the container has to be greater than atmospheric pressure.**
2. **Pressure must be relatively small to maintain a small column of fluid.**
3. **The measurement of pressure must be of a liquid.**

Note: $p_A = p_1$ because they are at the same level

Moving from left to right: $p_{A(abs)} - \gamma_1 h_1 = p_o$

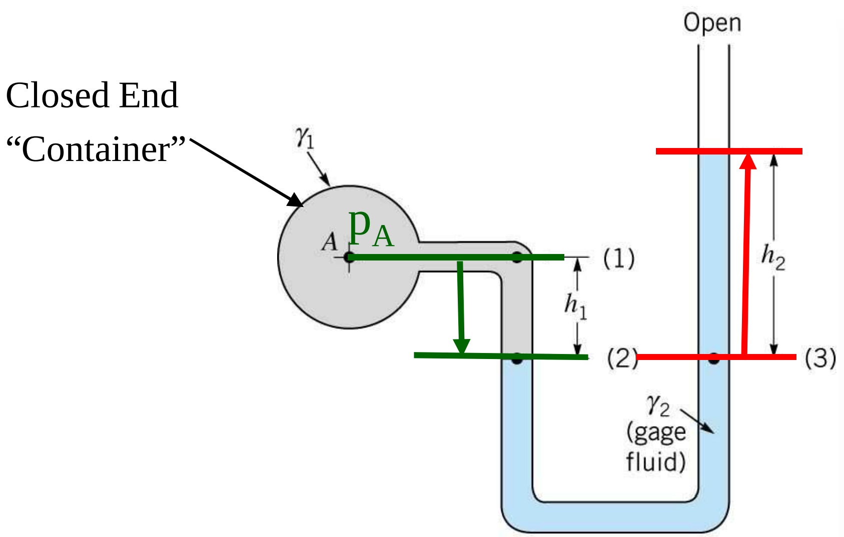
Rearranging: $p_A - p_o = \gamma_1 h_1$

Gage Pressure

Then in terms of gage pressure, the equation for a Piezometer Tube:

$$p_A = \gamma_1 h_1$$

Measurement of Pressure: U-Tube Manometer



Note: in the same fluid we can "jump" across from 2 to 3 as they are at the sam level, and thus must have the same pressure.

The fluid in the U-tube is known as the gage fluid. The gage fluid type depends on the application, i.e. pressures attained, and whether the fluid measured is a gas or liquid.

Since, one end is open we can work entirely in gage pressure:

Moving from left to right: $p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$

Then the equation for the pressure in the container is the following:

$$p_A = \gamma_2 h_2 - \gamma_1 h_1$$

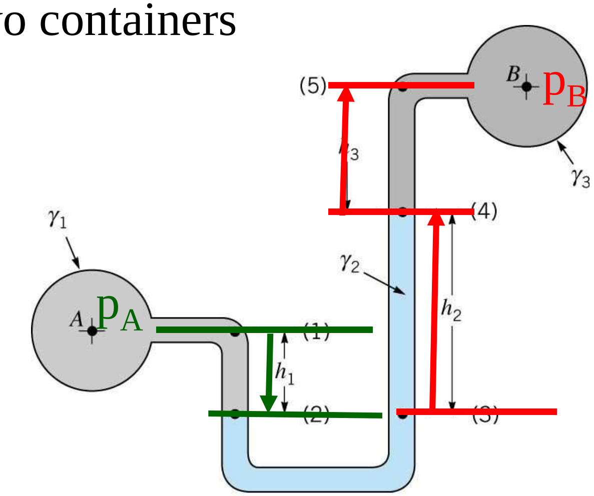
If the fluid in the container is a gas, then the fluid 1 terms can be ignored:

$$p_A = \gamma_2 h_2$$

Measurement of Pressure: U-Tube Manometer

➢Measuring a Pressure Differential *between* two containers

Closed End "Container"



Closed End "Container"

Final notes:

1. Common gage fluids are Hg and Water, some oils, and must be immiscible.
2. Temp. must be considered in very accurate measurements, as the gage fluid properties can change.
3. Capillarity can play a role, but in many cases each meniscus will cancel.

Meniscus: The curved free surface of a liquid in a capillary tube

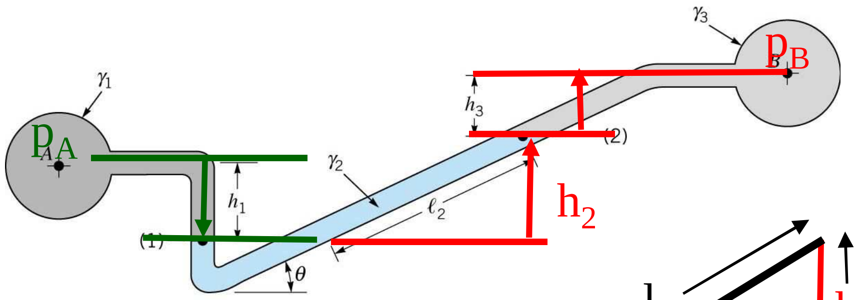
Moving from left to right: $p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$

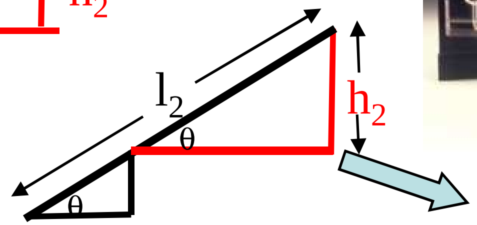
➢Then the equation for the pressure difference in the container is the following:

$$p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

Measurement of Pressure: Inclined-Tube Manometer

This type of manometer is used to measure small pressure changes.





$$\sin\theta = \frac{h_2}{l_2} \Rightarrow h_2 = l_2 \sin\theta$$

Moving from left to right: $p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$

Substituting for h_2: $p_A + \gamma_1 h_1 - \gamma_2 \ell_2 \sin\theta - \gamma_3 h_3 = p_B$

Rearranging to Obtain the Difference: $p_A - p_B = \gamma_2 \ell_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$

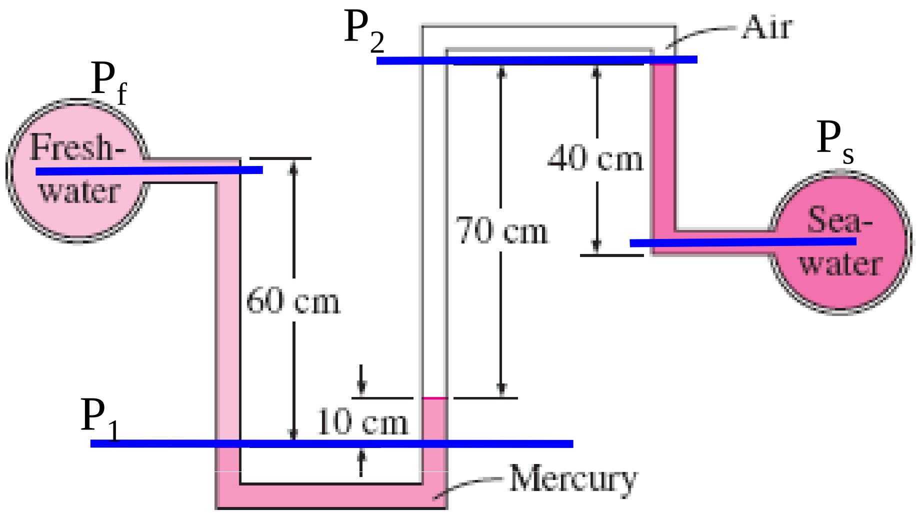
If the pressure difference is between gases: $p_A - p_B = \gamma_2 \ell_2 \sin\theta$

$$\ell_2 = \frac{p_A - p_B}{\gamma_2 \sin\theta}$$

Thus, for the length of the tube we can measure a greater pressure differential.

Example

- Freshwater and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer, as shown in. Determine the pressure difference between the two pipelines ($P_f - P_s$). Take the density of seawater at that location to be $\rho = 1035\ \text{kg/m}^3$



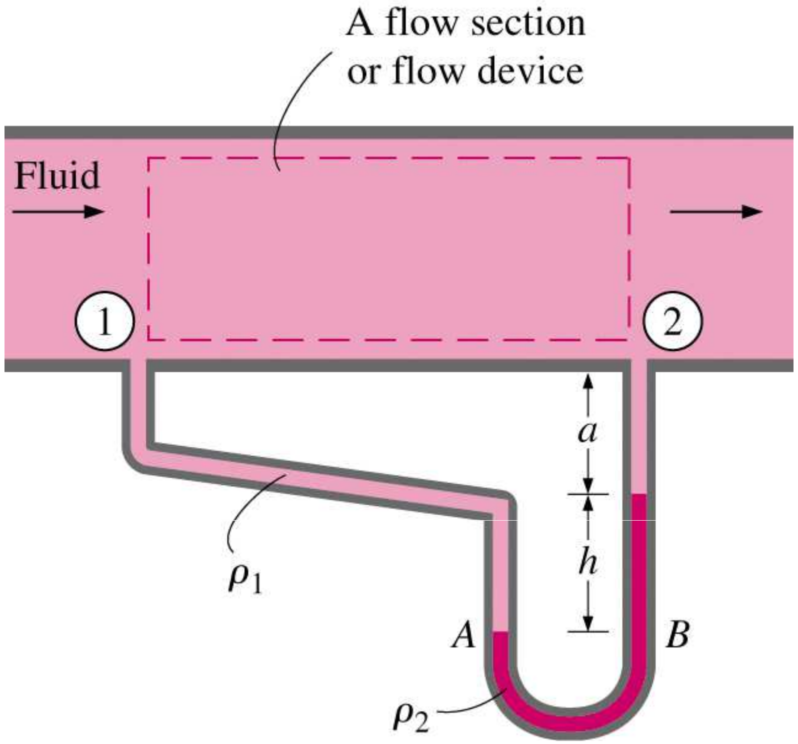
$$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \rho_{air} g h_{air} + \rho_{sea} g h_{sea} = P_2$$

$$= (9.81\ \text{m/s}^2)[(13600\ \text{kg/m}^3)(0.1\ \text{m})$$

$$-(1000\ \text{kg/m}^3)(0.6\ \text{m}) - (1035\ \text{kg/m}^3)(0.4\ \text{m})]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 3.39\ \text{kN/m}^2 = \mathbf{3.39\ kPa}$$

Measuring Pressure Drops



- Manometers are well-suited to measure pressure drops across valves, pipes, heat exchangers, etc.
- Relation for pressure drop P_1-P_2 is obtained by starting at point 1 and adding or subtracting ρgh terms until we reach point 2.
- If fluid in pipe is a gas, $\rho_2 >> \rho_1$ and P_1-P_2= ρgh

$$P_1 + \rho_1 g(a + h) - \rho_2 gh - \rho_1 ga = P_2$$

$$P_1 - P_2 = (\rho_2 - \rho_1)gh$$

When the fluid flowing in the pipe is a gas $\rho_1 \ll \rho_2$

$$P_1 - P_2 \approx \rho_2 gh$$

Example

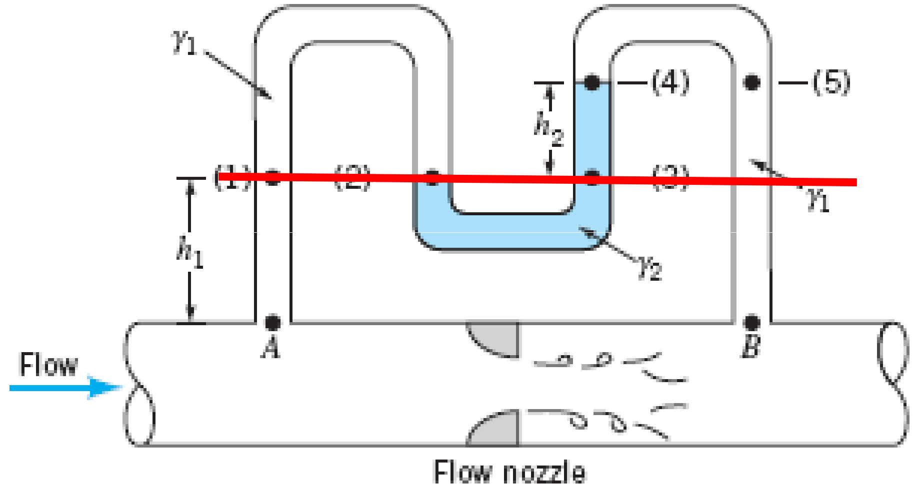
In the figure (a) Determine an equation for $p_A - p_B$ in terms of the specific weight of the flowing fluid, γ_1, the specific weight of the gage fluid, γ_2, and the various heights indicated. (b) For $\gamma_1 = 9.80\ \text{kN/m}^3$, $\gamma_2 = 15.6\ \text{kN/m}^3$, $h_1 = 1.0$ m, and $h_2 = 0.5$ m, what is the value of the pressure drop, $p_A - p_B$?

$$p_A - \gamma_1 h_1 - \gamma_2 h_2 + \gamma_1(h_1 + h_2) = p_B$$

$$p_A - p_B = h_2(\gamma_2 - \gamma_1)$$

$$p_A - p_B = (0.5\ \text{m})(15.6\ \text{kN/m}^3 - 9.80\ \text{kN/m}^3)$$

$$= 2.90\ \text{kPa}$$



Measurement of Pressure

Some common working fluids are

Liquid	Specific Gravity
Alcohol	0.75-0.87
Water	1.00
Mercury	13.6

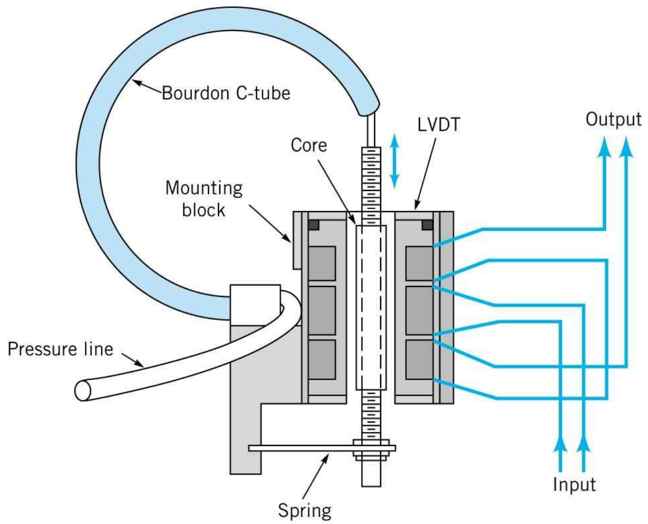
Although manometers are widely used,

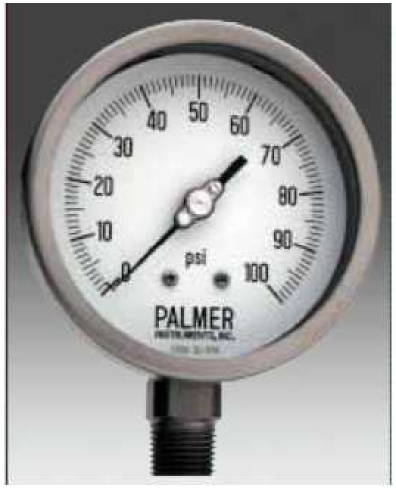
- They are not well suited for measuring very high pressures,
- or pressures that are changing rapidly with time.
- They require the measurement of one or more column heights, which, although not particularly difficult, can be time consuming.
- To overcome some of these problems numerous other types of pressure measuring instruments have been developed.

Measurement of Pressure: Mechanical and Electrical Devices

- Most of these types make use of the idea that when a pressure acts on an elastic structure the structure will deform,
- This deformation can be related to the magnitude of the pressure

Spring Bourdon Gage:





A good title of a report: Pressure measuring devices