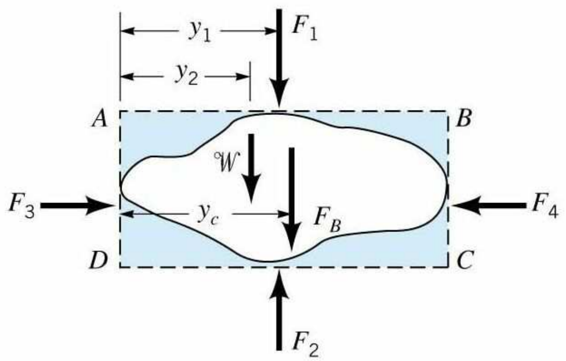
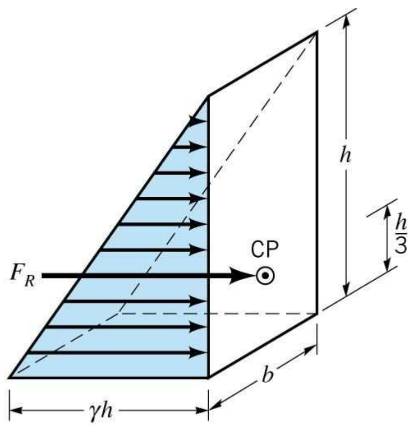
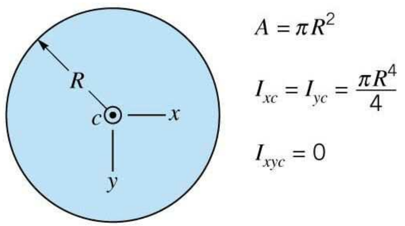
Fluid Mechanics

Pressure and Fluid Static (B)







Dr. Yousef Mubarak

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- Hydrostatic Force on a Plane Surface
- Pressure Prism
- Hydrostatic Force on a Curved Surface
- Buoyancy, Flotation, and Stability
- Rigid Body Motion of a Fluid

Review

<u>We have seen the following features of statics fluids</u>

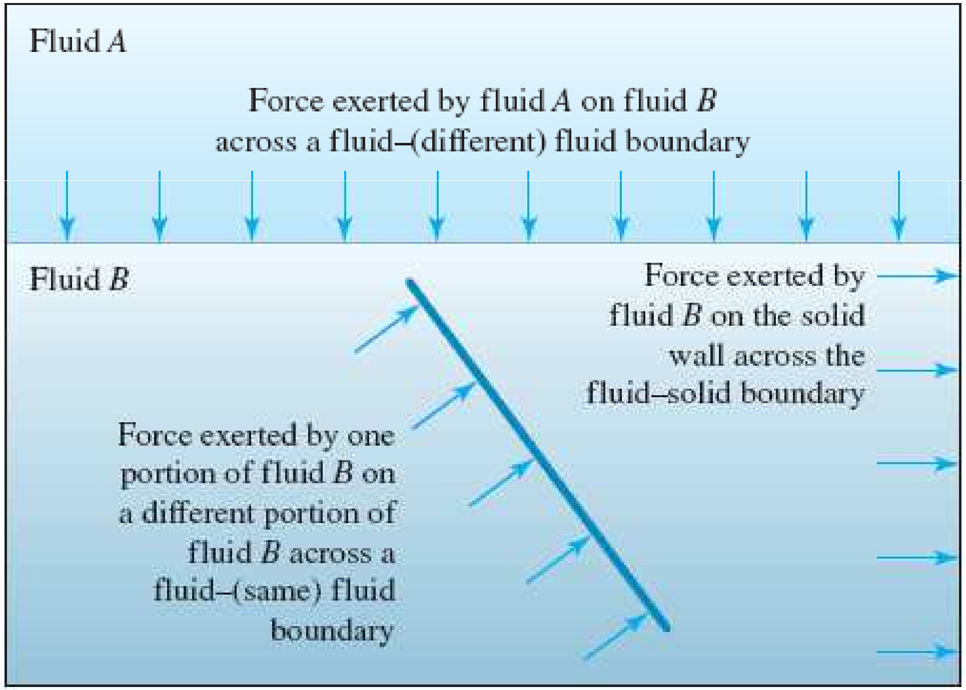
- Hydrostatic vertical pressure distribution
- Pressures at any equal depths in a continuous fluid are equal.
- Pressure at a point acts equally in all directions (Pascal. s law).
- For fluids at rest we know that the force must be *perpendicular* to the surface since there are no shearing stresses present.
- The pressure will vary linearly with depth if the fluid is incompressible.

Classification of Fluid Forces

All forces in fluid mechanics are divided into two distinctive types:

i. **Body forces** (gravity , centrifugal and electromagnetic forces), are external forces that act on a small fluid element in such a way that the magnitude of the body force is proportional to the element's mass (or volume) $$(\delta M_j)_{\Re} = (\delta m)_{\Re}\, g_j = \rho \delta V_{\Re} g_j$$

- Body forces are considered to be external forces; i.e., they are thought of as acting on a fluid, but not as forces applied by a fluid
- They exert their influence on a fluid at rest or in motion without the need for physical contact between the external source of the body force and the fluid

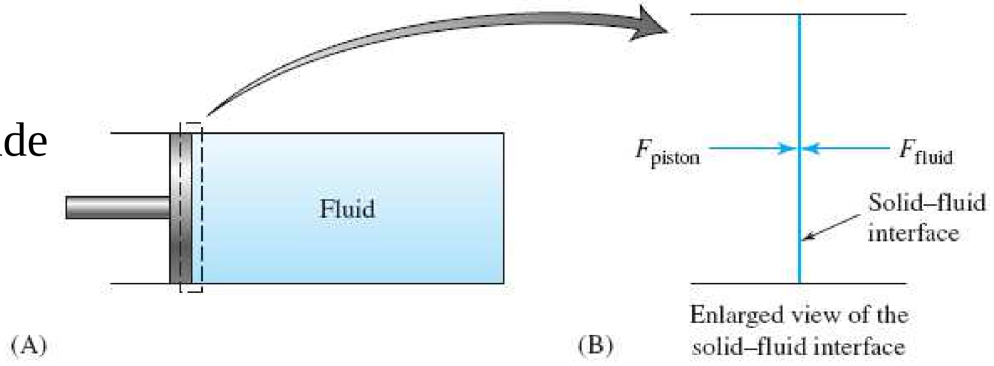


Classification of Fluid Forces

ii. **Surface forces,** such as those exerted by pressure or shear stress, are forces that act on a fluid element through physical contact between the element and its surroundings.

- Surface force is exerted across every boundary or interface between a fluid and another material.
- The second material may be a solid, another portion of the same fluid, or a different fluid.
- These forces exist at every interface wetted by a fluid and are present irrespective of whether the fluid is at rest (pressure only) or in motion (pressure and shear stress).
- From a macroscopic point of view, surface forces are applied to this interface by the piston and by the fluid are equal in magnitude and opposite in direction (Newton's second law).

$$(\delta O_j)_{\Re} = -\frac{\partial P}{\partial x_j}\delta V_{\Re}.$$



Classification of Fluid Forces

- From a molecular perspective, the surface forces applied to this interface by the fluid are generated through molecular momentum transfer through the interaction of moving fluid molecules with the molecules of the solid piston.

- Surface forces are the macroscopic consequence of molecular momentum transfer

- Surface forces are forces exerted by a fluid on a solid is always equal and opposite to the force exerted by a solid on a fluid.

- A surface force depends on the contact area between a fluid and a second material.

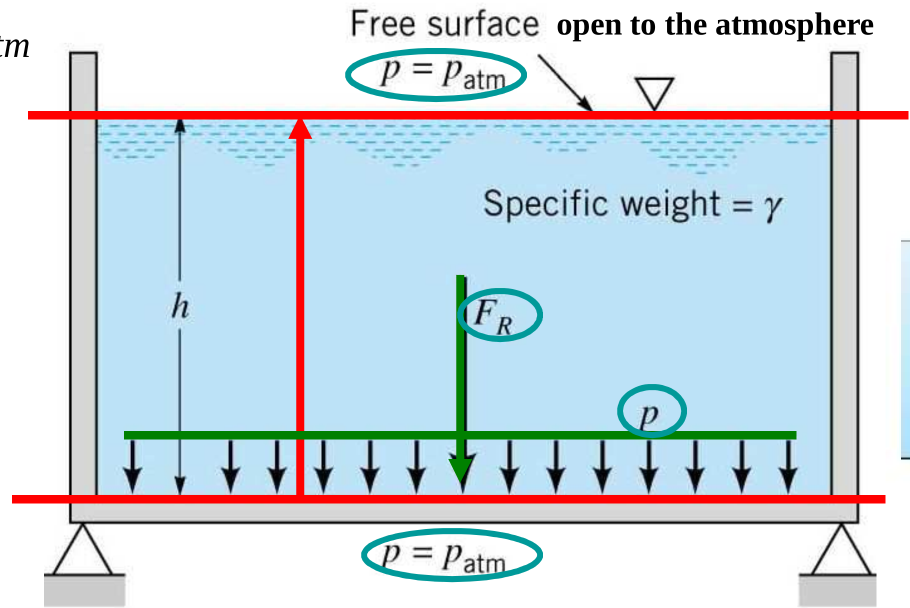
Hydrostatic Force on a Plane Surface: Tank Bottom

- When a surface is submerged in a fluid, forces develop on the surface due to the fluid.
- For fluid at rest, force must be *perpendicular* to the surface since there are no shearing stresses present.

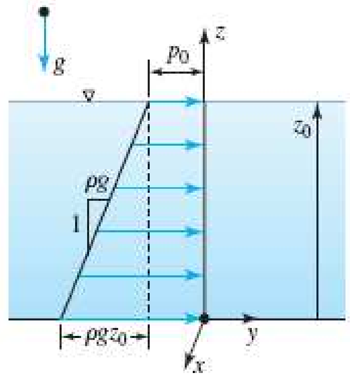
Simplest Case: Tank bottom with a uniform pressure distribution

$$p - \gamma h = p_{atm} - p_{atm}$$

$$p = \gamma h$$



For incompressible



Now, the resultant Force:

$$F_R = pA$$

Acts through the Centroid

A = area of the Tank Bottom

Hydrostatic Force on a Plane Surface: General Case

- **Determine the direction, location, and magnitude of the resultant force acting on an submerged inclined plane surface.**

The origin O is at the Free Surface.

θ is the angle the plane makes with the free surface.

y is directed along the plane surface.

A is the area of the surface.

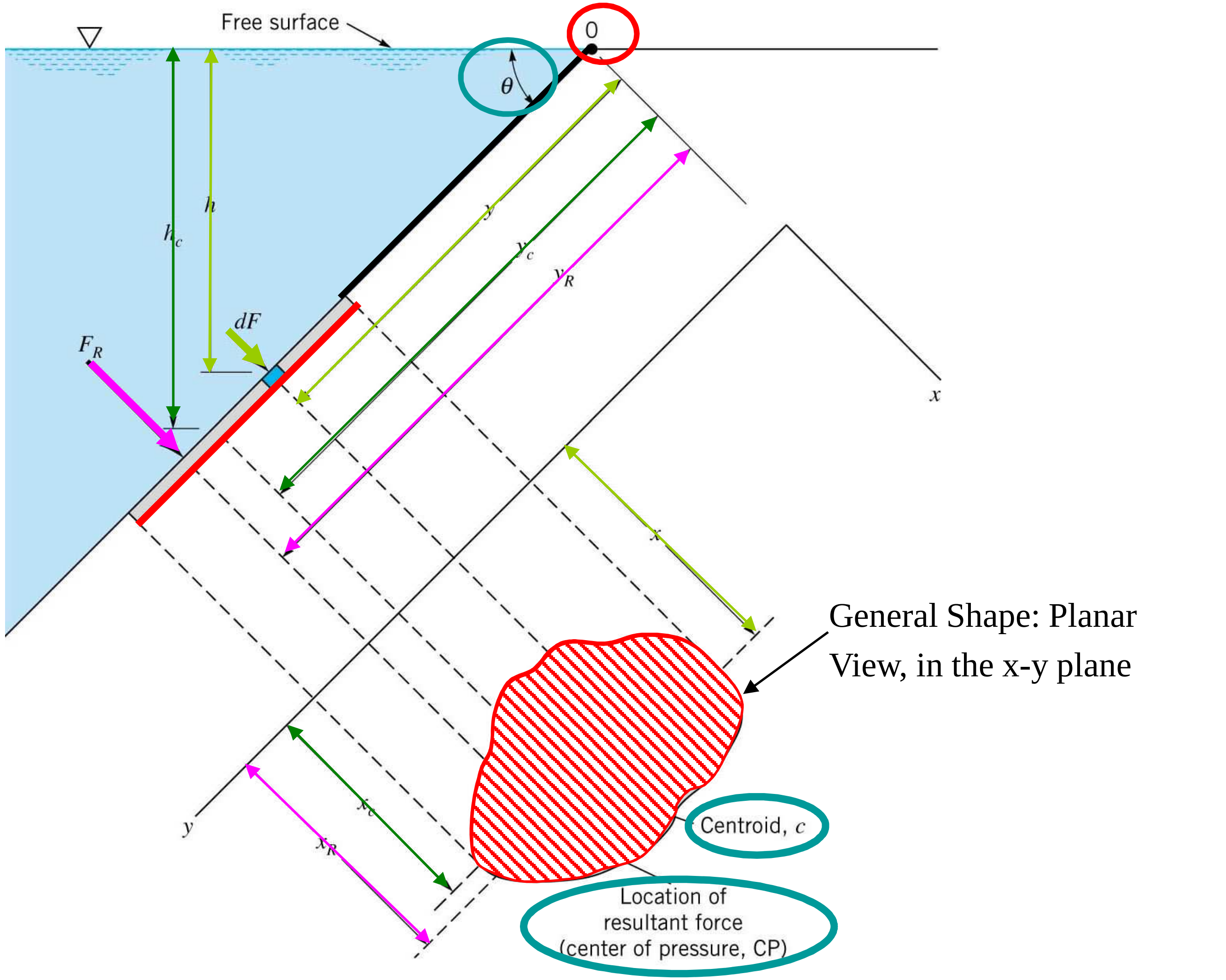
dA is a differential element of the surface.

dF is the force acting on the differential element.

C is the centroid.

CP is the center of Pressure

F_R is the resultant force acting through CP

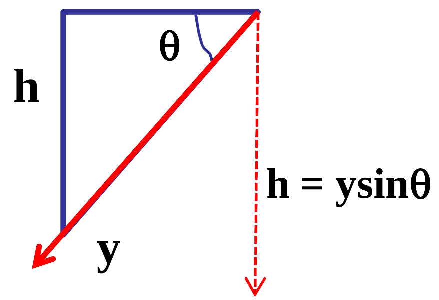


Hydrostatic Force on a Plane Surface: General Case

Then the force acting on the differential element:

$$dF = \gamma h \, dA$$

Then the resultant force acting on the entire surface:



$$F_R = \int_A \gamma h \, dA$$

We note h = ysinθ

$$= \int_A \gamma y \sin\theta \, dA$$

With γ and θ taken as constant:

$$F_R = \gamma \sin\theta \int_A y \, dA$$

We note, the integral part is the first moment of area about the x-axis

$$\int_A y \, dA = y_c A$$

Where y_c is the y coordinate to the centroid of the object.

$$F_R = \gamma A \underbrace{y_c \sin\theta}_{h_c} \longrightarrow \boxed{F_R = \gamma h_c A}$$

where h_c is the vertical distance from the fluid surface to the centroid of the area

Hydrostatic Force on a Plane Surface

- The magnitude of the force is independent of the angle θ and depends only on the specific weight of the fluid, the total area, and the depth of the centroid of the area below the surface
- The magnitude of the resultant force is equal to the pressure at the centroid of the area multiplied by the total area.
- Since all the differential forces that were summed to obtain F_R are perpendicular to the surface, the resultant F_R must also be perpendicular to the surface
- The location of the force F_R *(y_R)* can be determined by summation of moments around the *x axis, that is*, the moment of the resultant force must equal the moment of the distributed pressure force, or

$$F_R y_R = \int_A y \, dF = \int_A \gamma \sin\theta \, y^2 \, dA \qquad \text{where} \qquad F_R = \gamma A y_c \sin\theta$$

Hydrostatic Force on a Plane Surface

$$y_R = \frac{\int_A y^2 \, dA}{y_c A}$$

Second moment of the area, I_{xc}

$$\boxed{y_R = \frac{I_{xc}}{y_c A} + y_c}$$

- Where I_{xc} is the second moment of the area with respect to an axis passing through its *centroid* and parallel to the *x axis*.

- **For a submerged plane, the resultant force always acts below the centroid of the plane.**

$$I_{xc}/y_c A > 0.$$

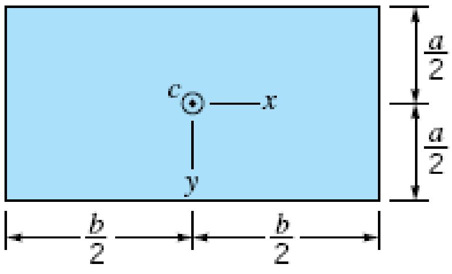
- Similarly, the x-coordinate can be found

$$F_R x_R = \int_A \gamma \sin\theta \, xy \, dA \;\Rightarrow\; x_R = \frac{\int_A xy \, dA}{y_c A} = \frac{I_{xy}}{y_c A} \;\Rightarrow\; \boxed{x_R = \frac{I_{xyc}}{y_c A} + x_c}$$

Keep in mind that $h_c = y_c \sin\theta$

Hydrostatic Force on a Plane Surface

Geometric properties of some common shapes.

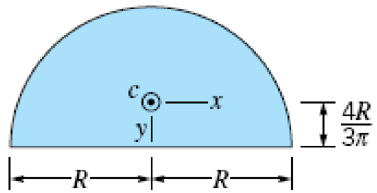


$A = ba$

$I_{xc} = \frac{1}{12} ba^3$

$I_{yc} = \frac{1}{12} ab^3$

$I_{xyc} = 0$

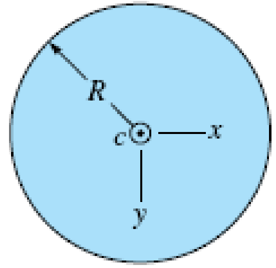


$A = \frac{\pi R^2}{2}$

$I_{xc} = 0.1098R^4$

$I_{yc} = 0.3927R^4$

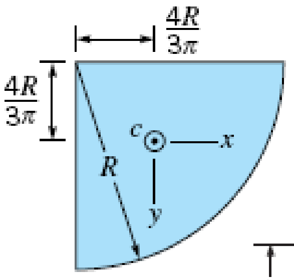
$I_{xyc} = 0$



$A = \pi R^2$

$I_{xc} = I_{yc} = \frac{\pi R^4}{4}$

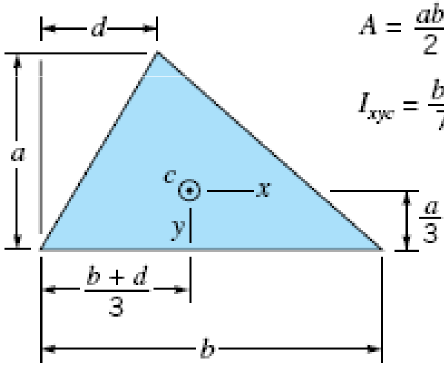
$I_{xyc} = 0$



$A = \frac{\pi R^2}{4}$

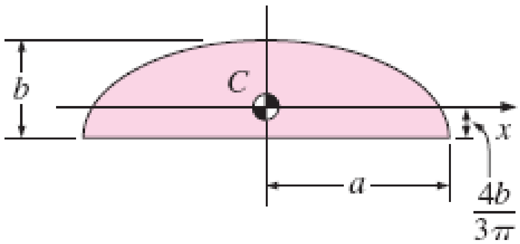
$I_{xc} = I_{yc} = 0.05488R^4$

$I_{xyc} = -0.01647R^4$

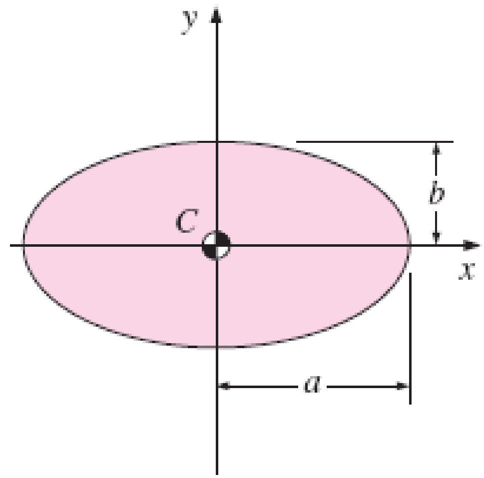


$A = \frac{ab}{2}$ $\quad I_{xc} = \frac{ba^3}{36}$

$I_{xyc} = \frac{ba^2}{72}(b - 2d)$



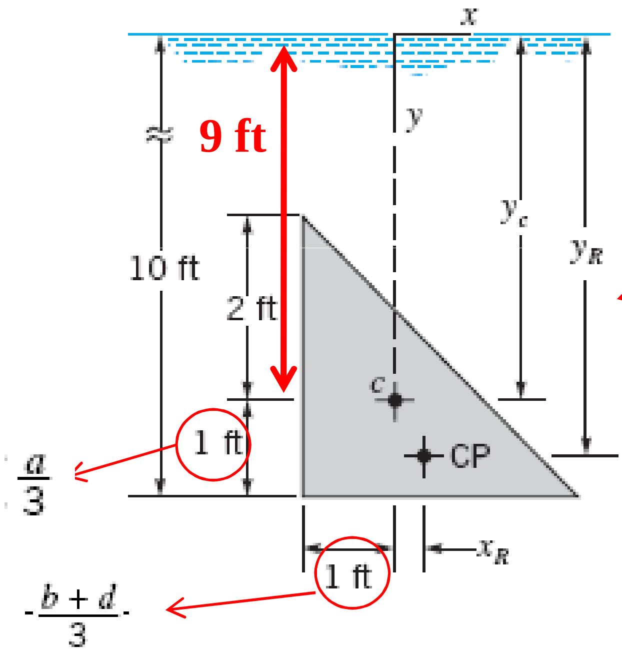
$A = \pi ab/2,\ I_{xx,\,C} = 0.109757ab^3$

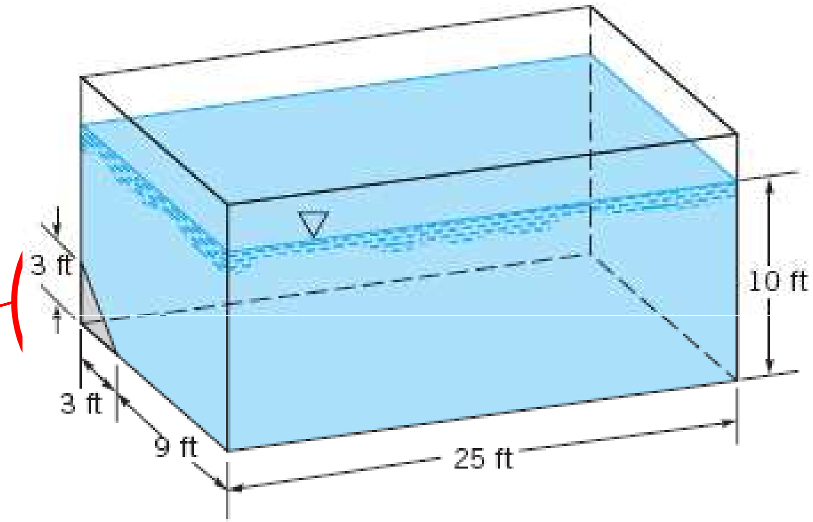


$A = \pi ab,\ I_{xx,\,C} = \pi ab^3/4$

Example

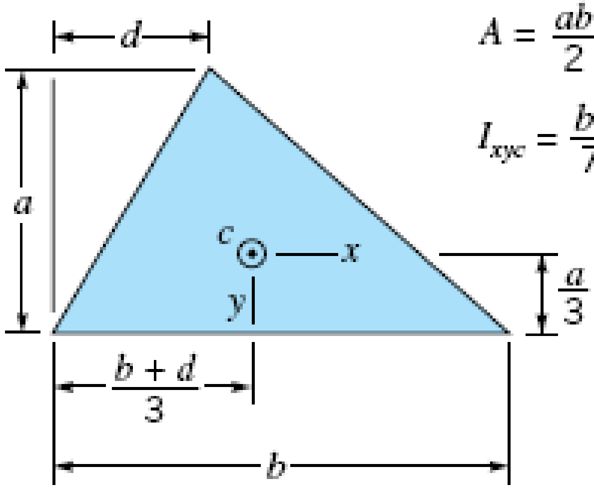
- A large fish-holding tank contains seawater $(\gamma = 64.0 \text{ lb/ft}^3)$ to a depth of 10 ft as shown. To repair some damage to one corner of the tank, a triangular section is replaced with a new section as illustrated. Determine the magnitude and location of the force of the seawater on this triangular area.





$$h_c = y_c \sin\theta = y_c \sin(90) = y_c$$

= 10-1 = 9 ft



$$A = \frac{ab}{2} \qquad I_{xc} = \frac{ba^3}{36}$$

$$I_{xyc} = \frac{ba^2}{72}(b - 2d)$$

$$\boxed{F_R = \gamma h_c A} \quad = (64.0\ \text{lb/ft}^3)(9\ \text{ft})(9/2\ \text{ft}^2) = 2590\ \text{lb}$$

A= ½*b*a

The y coordinate of the center of pressure (CP) is found from

$$y_R = \frac{I_{xc}}{y_c A} + y_c \qquad \text{where} \qquad I_{xc} = \frac{ba^3}{36}$$

$$I_{xc} = \frac{(3\ \text{ft})(3\ \text{ft})^3}{36} = \frac{81}{36}\text{ft}^4$$

$$y_R = \frac{81/36\ \text{ft}^4}{(9\ \text{ft})(9/2\ \text{ft}^2)} + 9\ \text{ft} = 0.0556\ \text{ft} + 9\ \text{ft} = 9.06\ \text{ft}$$

➢The x coordinate of the center of pressure $x_R = \dfrac{I_{xyc}}{y_c A} + x_c$ where $I_{xyc} = \dfrac{ba^2}{72}(b - 2d)$

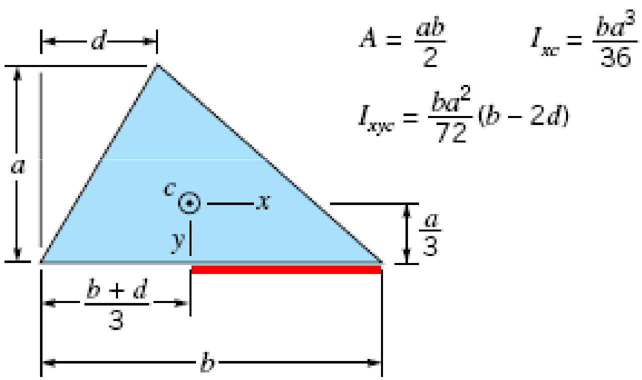
$$I_{xyc} = \frac{(3\ \text{ft})(3\ \text{ft})^2}{72}(3\ \text{ft}) = \frac{81}{72}\text{ft}^4$$

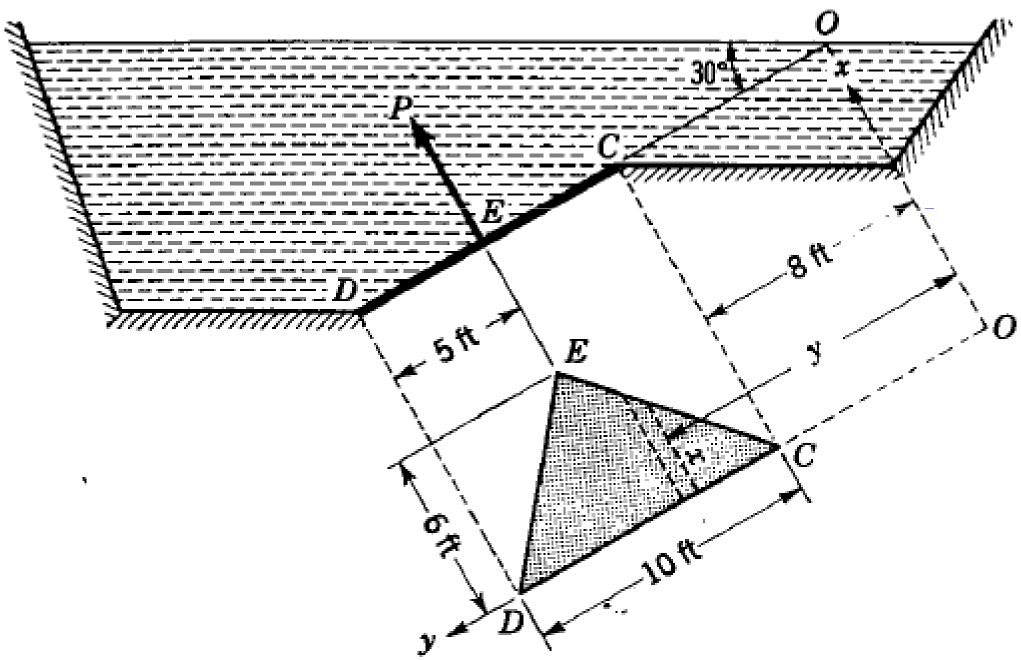
$$x_R = \frac{81/72\ \text{ft}^4}{(9\ \text{ft})(9/2\ \text{ft}^2)} + 0 = 0.0278\ \text{ft}$$

Thus, we conclude that the center of pressure is 0.0278 ft to the right of and 0.0556 ft below the centroid of the area

Triangular gate CDE is hinged along CD and is opened by a normal force F applied at E. It holds oil, s.g 0.80, above it and is open to the atmosphere on its lower side. Neglecting the weight of the gate determine (a) the magnitude of force exerted on the gate, (b) the location of pressure center; (c) the force P necessary to open the gate

= 5+8 =13 ft





$$b - \frac{b+d}{3} = 5$$

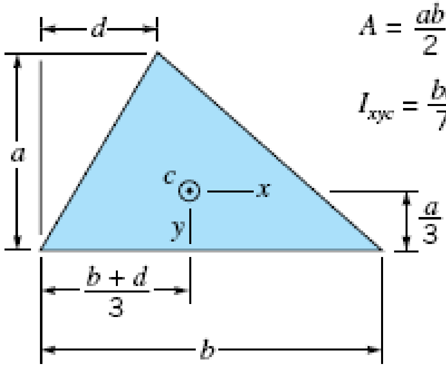
Example Cont.

$$F \doteq p_G A = \gamma y_c \sin\theta\, A = \underbrace{62.4}_{\gamma} \times \underbrace{0.80}_{s.g} \times \underbrace{0.50}_{\sin\theta} \times \underbrace{30}_{A} \times \underbrace{13}_{y_c} = 9734.4 \text{ lb}$$

$$x_R = \frac{\overset{= 0.0}{I_{xyc}}}{y_c A} + x_c \qquad \frac{a}{3} = 2$$

$$y_R = \frac{I_{xc}}{\underbrace{y_c A}_{= 0.15 \text{ ft}}} + \underbrace{y_c}_{= 13 \text{ ft}}$$

$y_R = 13.15$ ft



$$A = \frac{ab}{2} \qquad I_{xc} = \frac{ba^3}{36}$$

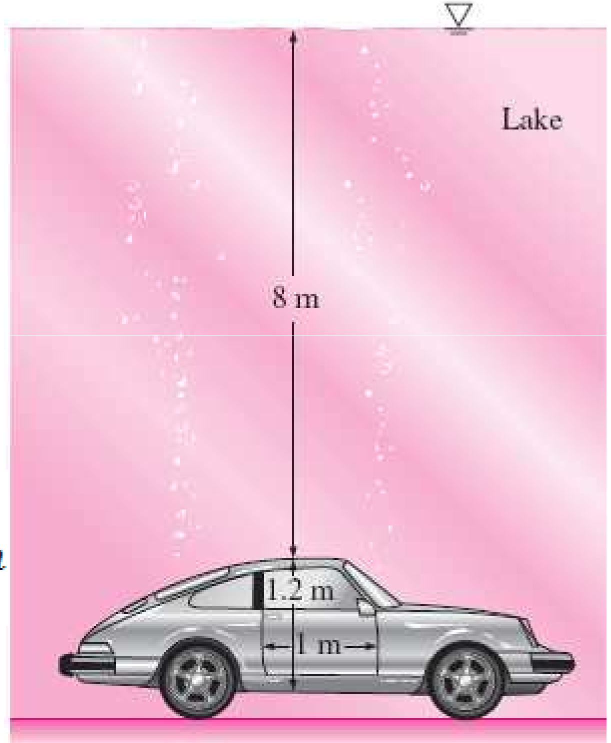
$$I_{xyc} = \frac{ba^2}{72}(b - 2d)$$

When moments about CD are taken and the action of the oil is replaced by the resultant,

$$P \times 6 = 9734.4 \times 2 \qquad P = 3244.8 \text{ lb}$$

Example

➢ A heavy car plunges into a lake during an accident and lands at the bottom of the lake on its wheels. The door is 1.2 m high and 1 m wide, and the top edge of the door is 8 m below the free surface of the water. Determine the hydrostatic force on the door and the location of the pressure center, and discuss if the driver can open the door.



$$F_R = \gamma A y_c \sin\theta$$

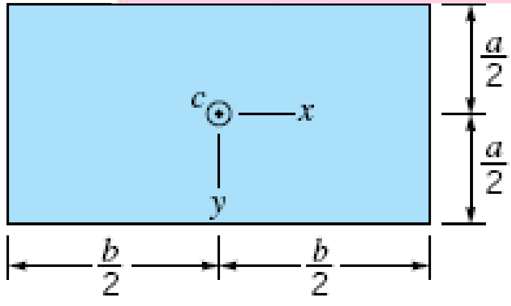
$$\boxed{F_R = \gamma h_c\, A}$$

$$h_c = 8 + \frac{a}{2} = 8.6\ m$$

$$F_R = 1000(9.81)(8.6) * (1 * 1.2) = 101.3\ KN$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c \quad \Rightarrow \quad y_R = \frac{\frac{1}{12} * (1)(1.2)^3}{8.6(1.2)(1)} + 8.6 = 8.614\ m$$

$$x_R = 0.0$$



$A = ba$

$I_{xc} = \frac{1}{12} ba^3$

$I_{yc} = \frac{1}{12} ab^3$

$I_{xyc} = 0$

Hydrostatic Force: Vertical Wall

Find the Pressure on a Vertical Wall using Hydrostatic Force Method

Pressure varies linearly with depth by the hydrostatic equation:

The magnitude of pressure at the surface p_o = 0.0 and at bottom is $p = \gamma h$

The depth of the fluid is "h"

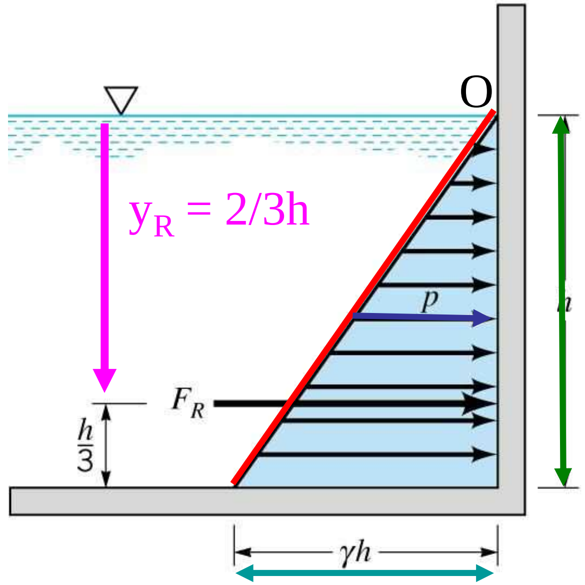
The width of the wall is "b" into the board

$$F_R = p_{av} A$$

By inspection, the average pressure occurs at h/2, $p_{av} = \gamma h/2$

$$F_R = \gamma \left(\frac{h}{2}\right) A$$

The resultant force act through the center of pressure, CP:



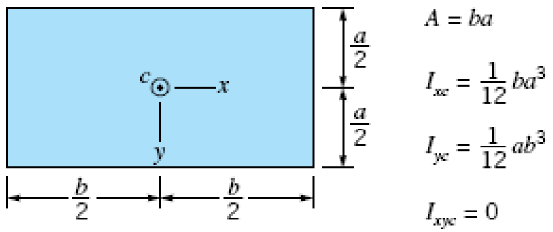
y-coordinate:

$$\boxed{y_R = \frac{I_{xc}}{y_c A} + y_c}$$

$$I_{xc} = \frac{1}{12} b h^3 \qquad y_c = \frac{h}{2} \qquad A = bh$$

$$y_R = \frac{bh^3}{12 \frac{h}{2}(bh)} + \frac{h}{2}$$

$$y_R = \frac{h}{6} + \frac{h}{2} = \frac{2}{3} h$$



Hydrostatic Force: Vertical Wall

x-coordinate:

$$\boxed{x_R = \frac{I_{xyc}}{y_c A} + x_c}$$

$$I_{xyc} = 0$$

$$x_c = \frac{b}{2}$$

$$A = bh$$

$$\Rightarrow$$

$$x_R = \frac{0}{\frac{h}{2}(bh)} + \frac{b}{2}$$

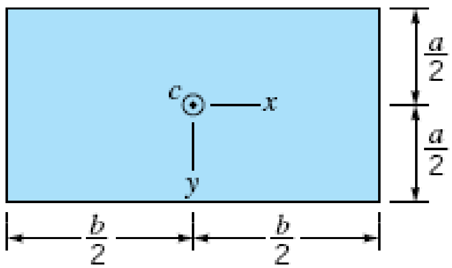
$$x_R = \frac{b}{2}$$

Center of Pressure:

$$\left(\frac{b}{2}, \frac{2h}{3}\right)$$

Example

➢ A pressurized tank contains oil $(SG = 0.90)$ and has a square, 0.6-m by 0.6-m plate bolted to its side, as is illustrated. *When the pressure gage on the top of the tank reads* 50 kPa, what is the magnitude and location of the resultant force on the attached plate? Given that the outside of the tank is at atmospheric pressure.

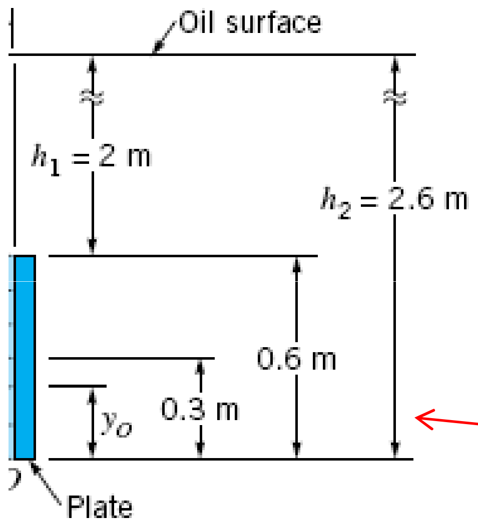


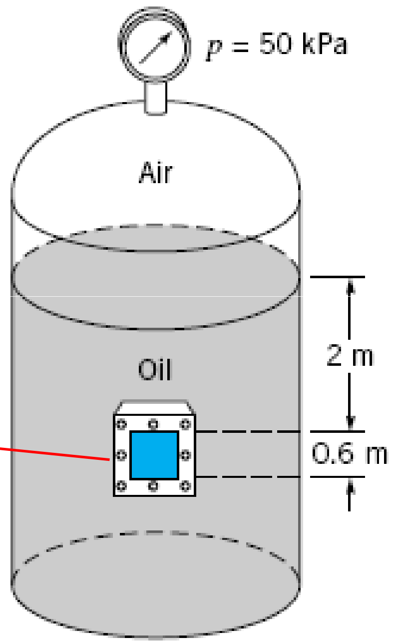
$A = ba$

$I_{xc} = \frac{1}{12} ba^3$

$I_{yc} = \frac{1}{12} ab^3$

$I_{xyc} = 0$





$$F_1 = (p_s + \gamma h_1) A$$

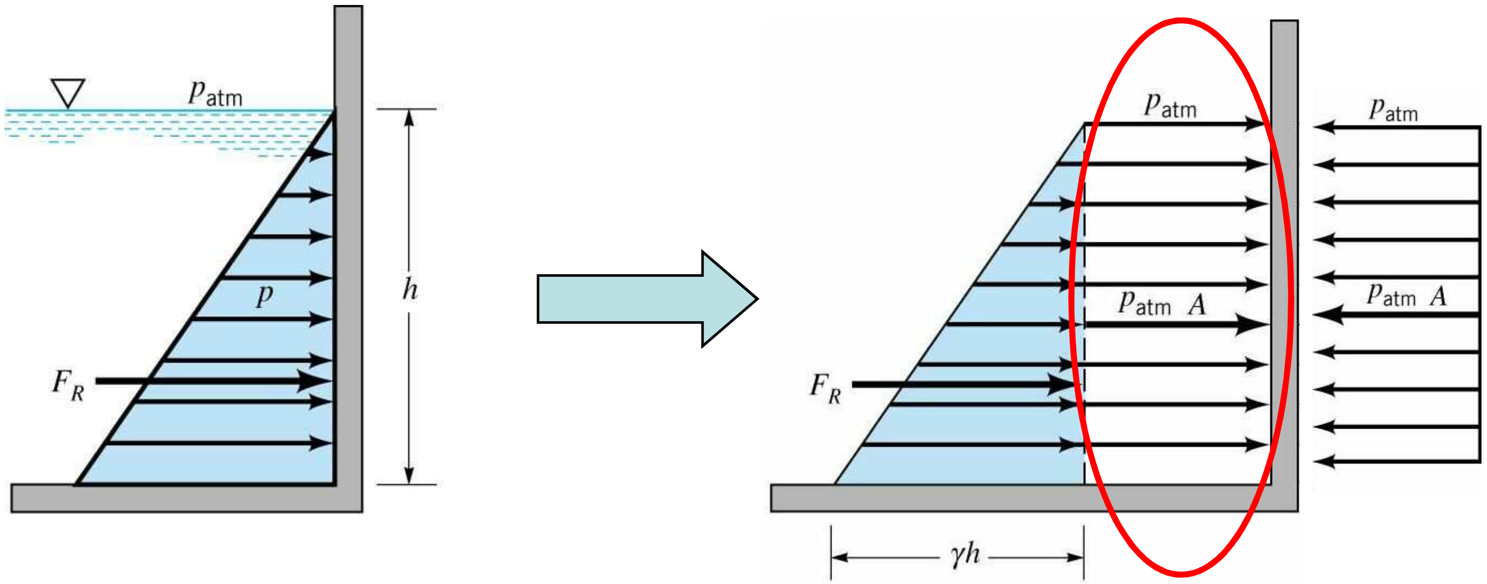
$$= [50 \times 10^3\ \text{N/m}^2 + (0.90)(9.81 \times 10^3\ \text{N/m}^3)(2.3\ \text{m})](0.36\ \text{m}^2)$$

$$= 25.4\ \text{kN}$$

$$\boxed{y_R = \frac{I_{xc}}{y_c A} + y_c} \longrightarrow y_R = 2.313$$

Atmospheric Pressure on a Vertical Wall

Gage Pressure Analysis   Absolute Pressure Analysis   But,

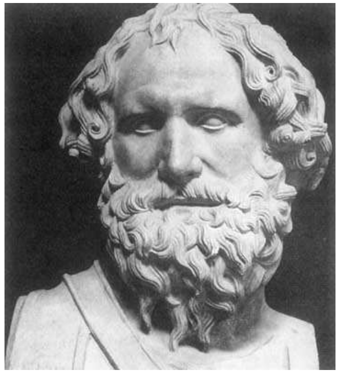


So, in this case the resultant force is the same as the gage pressure analysis.

It is not the case if the container is closed with a vapor pressure above it.

If the plane is submerged, there are multiple possibilities.

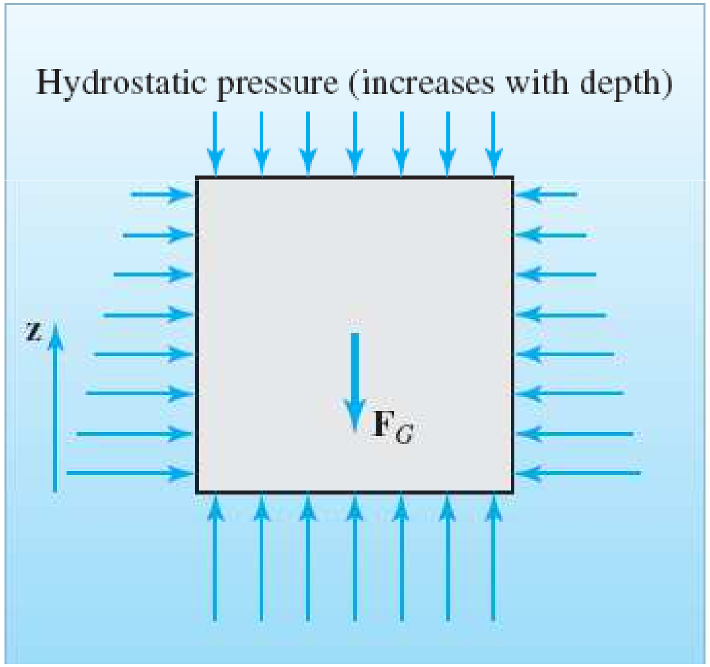
Buoyancy: Archimedes' Principle



Archimedes (287-212 BC)

Archimedes' Principle states that the buoyant force has a magnitude equal to the weight of the fluid displaced by the body and is directed vertically upward through the centroid of the displaced volume.

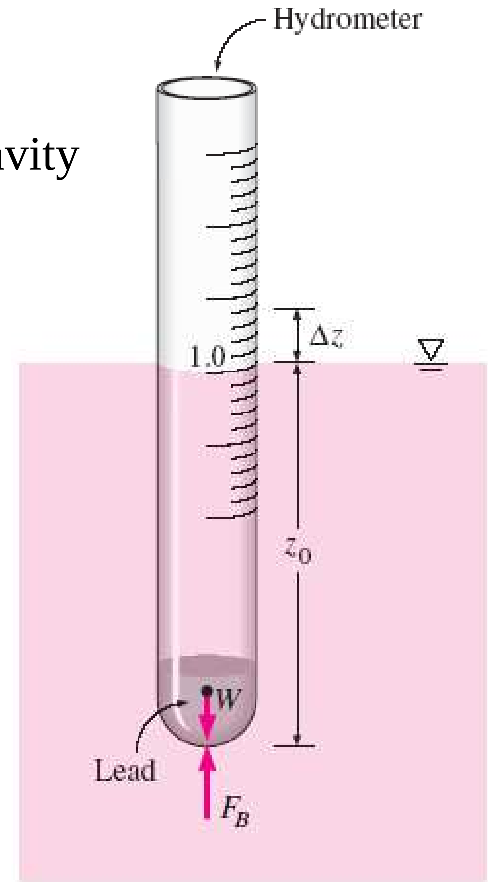
- Buoyant force is a force that results on a floating or submerged body in a fluid.
- The force results from different pressures on the top and bottom of the object
- The pressure forces acting from below are greater than those on top



Buoyancy: Archimedes' Principle

- The altitude of a hot air balloon is controlled by the temperature difference between the air inside and outside the balloon, since warm air is less dense than cold air. When the balloon is neither rising nor falling, the upward buoyant force exactly balances the downward weight.

Measuring Specific Gravity by a Hydrometer



$$SG_f = \frac{\rho_f}{\rho_w} = \frac{z_0}{z_0 + \Delta z}$$

Show this

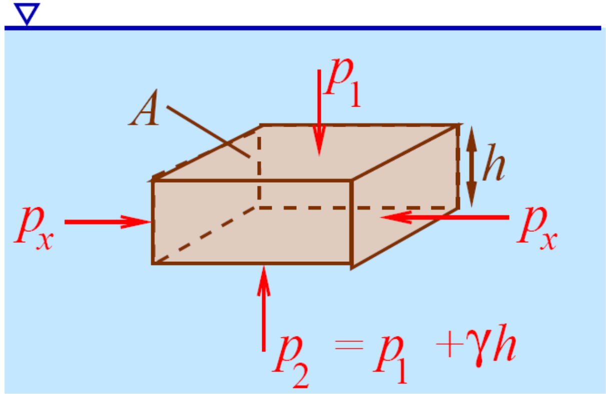
Buoyancy and Flotation: Archimedes' Principle

Balancing the Forces of the F.B.D. in the vertical Direction:

$$F_{Hydro} = p_2 A - p_1 A$$

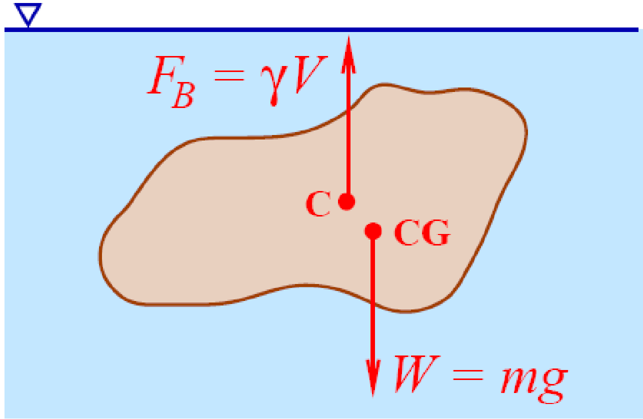
$$F_{Hydro} = (p_1 A + \gamma h) A - p_1 A$$

$$F_{Hydro} = \gamma h A = \gamma V = F_b$$



- The net hydrostatic force of the fluid on the body is vertically upward and is known as the Buoyant Force. The force is equal to the weight of the fluid it displaces.
- The Buoyancy force of a submerged body passes through a centroid called the center of buoyancy
- The weight force passes through the center of gravity and does not always pass through the buoyancy centroid.

$$F_{net} = (\rho_H - \rho_C)\forall g$$

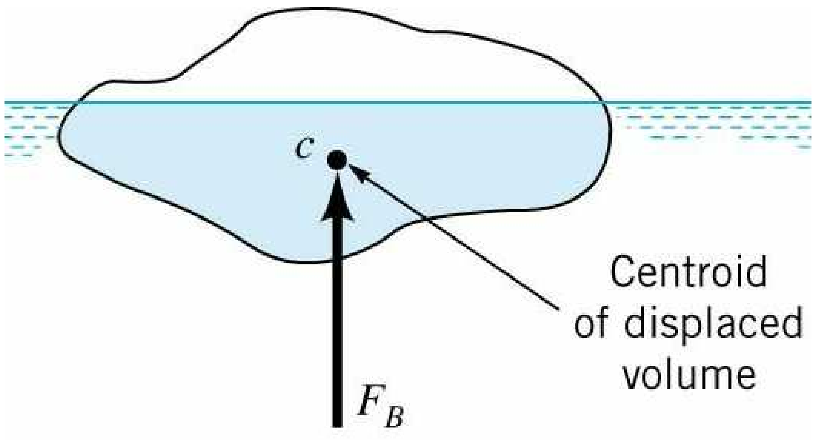


$$F_{net} = F_{gravity} - F_{buoyancy} = W_{obj} - W_{fluid}$$

$$\mathbf{F_{net} = (\rho_{obj} - \rho_F)\,Vg}$$

Buoyancy and Flotation: Archimedes' Principle

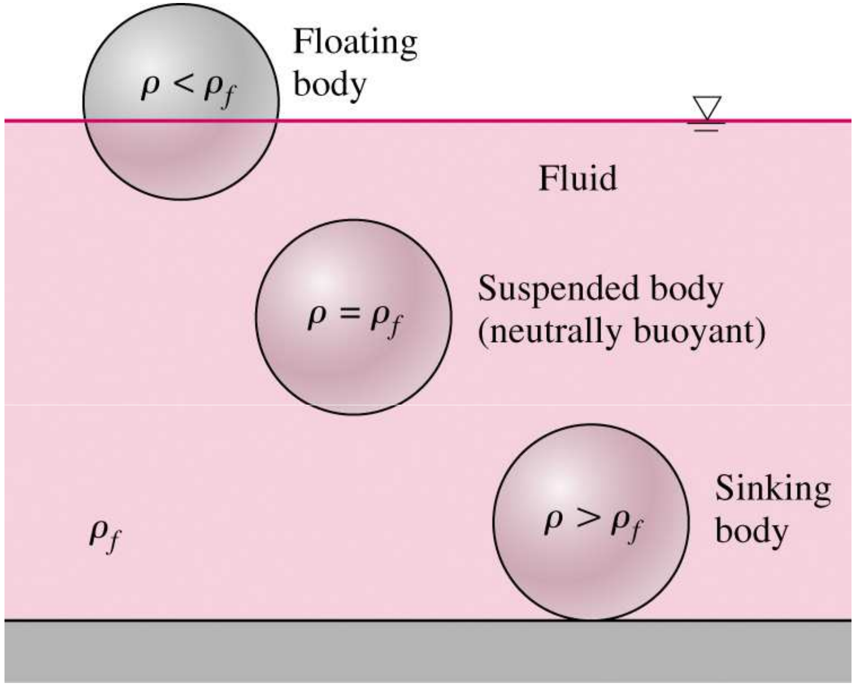
We can apply the same principles to floating objects:



- Conclude that the *buoyant force passes through the centroid of the displaced volume as shown*.

- If the specific weight varies in the fluid, the buoyant force does not pass through the centroid of the displaced volume, but through the center of gravity of the displaced volume.

Buoyancy and Flotation: Archimedes' Principle



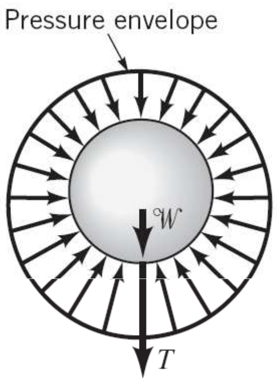
- Buoyancy force F_B is equal only to the displaced volume $\rho_f g V_{displaced}$.
- Three scenarios possible
  1. $\rho_{body}<\rho_{fluid}$: Floating body
  2. $\rho_{body}=\rho_{fluid}$: Neutrally buoyant
  3. $\rho_{body}>\rho_{fluid}$: Sinking body

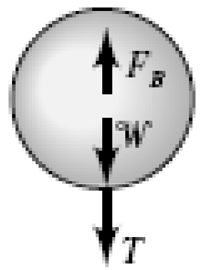
Example

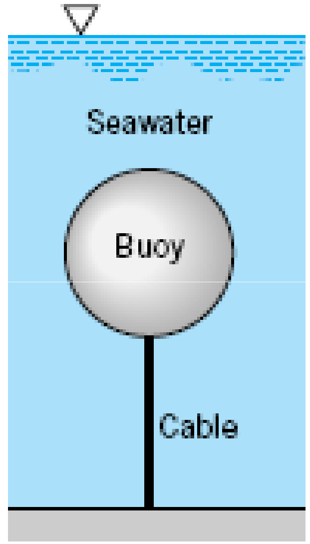
- A spherical buoy has a diameter of 1.5 m, weighs 8.50 kN, and is anchored to the sea floor with a cable as is shown. Although the buoy normally floats on the surface, at certain times the water depth increases so that the buoy is completely immersed as illustrated. For this condition what is the tension of the cable?

$$T = F_B - \mathcal{W}$$

$$F_B = \gamma \forall$$







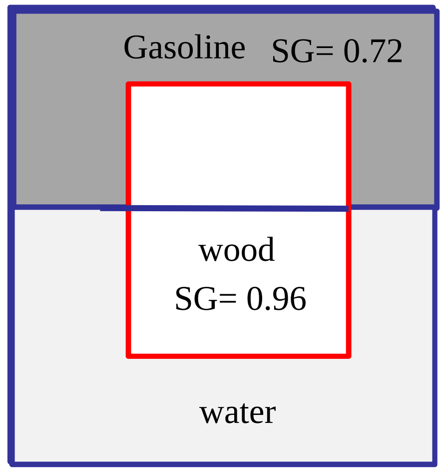
and for seawater with $\gamma = 10.1 \text{ kN/m}^3$ and $\forall = \pi d^3/6$ then

$$F_B = (10.1 \times 10^3 \text{ N/m}^3)[(\pi/6)(1.5 \text{ m})^3] = 1.785 \times 10^4 \text{ N}$$

$$T = 1.785 \times 10^4 \text{ N} - 0.850 \times 10^4 \text{ N} = 9.35 \text{ kN}$$

Example

- A block of wood floating at the interface between a layer of gasoline and a layer of water as shown. What fraction of the wood is below the interface?



Weight of the wood equal to the buoyant force, which is also equal to the two fluid displaced.

$$V_{wood}\rho_{wood}\, g = V_w \rho_w g + V_G \rho_G g$$

V_{wood}: *Volume of wood*

V_w *and* V_G *are the volume of water and gasoline displaced*

$$V_{wood} SG_{wood} = V_w + V_G SG_G$$

$$V_{wood} = V_w + V_G \quad \Longrightarrow \quad V_{wood} SG_{wood} = V_w + (V_{wood} - V_w) SG_G$$

$$\Longrightarrow \quad V_w \backslash V_{wood} = (\mathrm{SG}_{wood} - \mathrm{SG}_G) \backslash (1 - \mathrm{SG}_G) = 0.857$$