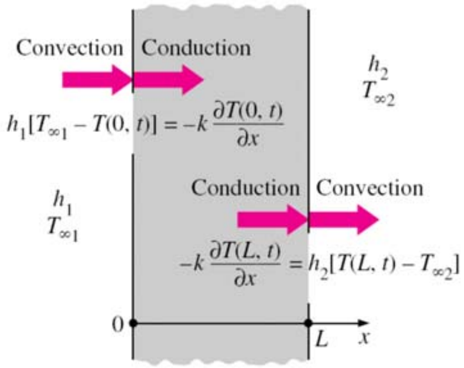
Convection Boundary Condition

Heat conduction at the surface in a selected direction = Heat convection at the surface in the same direction



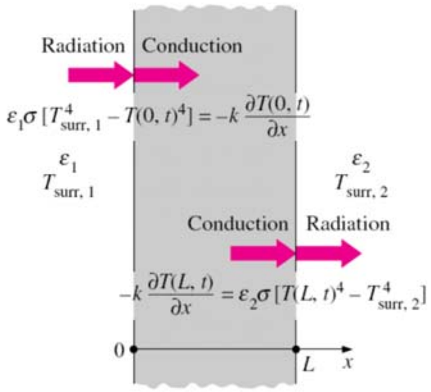
$$-k\frac{\partial T(0,t)}{\partial x} = h_1\left[T_{\infty 1} - T(0,t)\right]$$

and

$$-k\frac{\partial T(L,t)}{\partial x} = h_2\left[T(L,t) - T_{\infty 2}\right]$$

Radiation Boundary Condition

Heat conduction at the surface in a selected direction = Radiation exchange at the surface in the same direction



$$-k\frac{\partial T(0,t)}{\partial x} = \varepsilon_1\sigma\left[T^4_{surr,1} - T(0,t)^4\right]$$

and

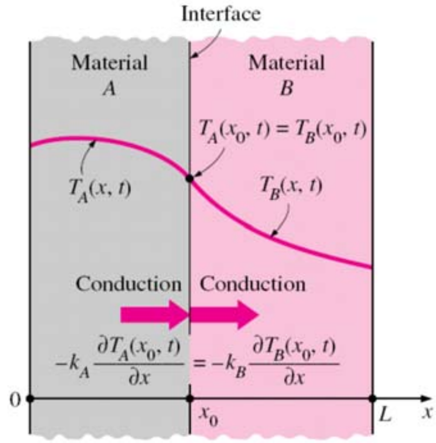
$$-k\frac{\partial T(L,t)}{\partial x} = \varepsilon_2\sigma\left[T(L,t)^4 - T^4_{surr,2}\right]$$

Interface Boundary Conditions

At the interface the requirements are:

1. two bodies in contact must have the *same temperature* at the area of contact,

(2) an interface (which is a surface) cannot store any energy, and thus the *heat flux* on the two sides of an interface *must be the same.*



$$T_A(x_0, t) = T_B(x_0, t)$$

and

$$-k_A \frac{\partial T_A(x_0,t)}{\partial x} = -k_B \frac{\partial T_B(x_0,t)}{\partial x}$$

Generalized Boundary Conditions

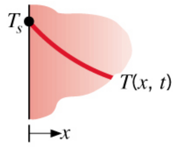
- In general a surface may involve convection, radiation, *and* specified heat flux simultaneously. The boundary condition in such cases is again obtained from a surface energy balance, expressed as

$$\begin{pmatrix} \text{Heat transfer} \\ \text{to the surface} \\ \text{in all modes} \end{pmatrix} = \begin{pmatrix} \text{Heat transfer} \\ \text{from the surface} \\ \text{In all modes} \end{pmatrix}$$

Boundary and Initial Conditions: summary

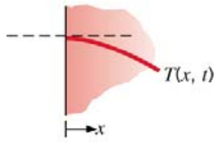
- For transient conduction, heat equation is first order in time, requiring specification of an initial temperature distribution: $T(x,t)_{t=0} = T(x,0)$
- Since heat equation is second order in space, two boundary conditions must be specified. Some common cases:

Constant Surface Temperature:



$$T(0,t) = T_s$$

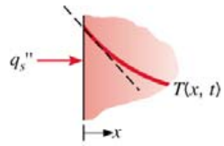
Insulated Surface



$$\frac{\partial T}{\partial x}\Big|_{x=0} = 0$$

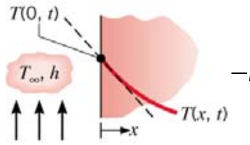
Constant Heat Flux:

Applied Flux



$$-k\frac{\partial T}{\partial x}\Big|_{x=0} = q_s''$$

Convection



$$-k\frac{\partial T}{\partial x}\Big|_{x=0} = h\left[T_\infty - T(0,t)\right]$$

Heat Generation in Solids -The Surface Temperature

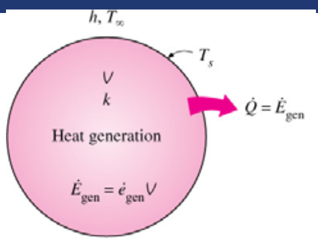


FIGURE 2–55
At steady conditions, the entire heat generated in a solid must leave the solid through its outer surface.

(Rate of heat transfer from the solid) = (Rate of energy generation within the solid)

For *uniform* heat generation within the medium

$$\dot{Q} = \dot{e}_{gen}V \qquad \text{(W)}$$

The heat transfer rate by convection can also be expressed from Newton's law of cooling as

$$\dot{Q} = hA_s\left(T_s - T_\infty\right) \qquad \text{(W)}$$

$$T_s = T_\infty + \frac{\dot{e}_{gen}V}{hA_s}$$

Heat Generation in Solids -The Surface Temperature

- For a large *plane wall* of thickness $2L$ (A_s=2A_{wall} and V=2LA_{wall})

$$T_{s,plane\ wall} = T_\infty + \frac{\dot{e}_{gen} L}{h}$$

- For a long solid *cylinder* of radius r_0 (A_s=2$\pi r_0 L$ and $V=\pi r_0^2 L$)

$$T_{s,cylinder} = T_\infty + \frac{\dot{e}_{gen} r_0}{2h}$$

- For a solid *sphere* of radius r_0 (A_s=4πr_0^2 and $V={}^4/_3\pi r_0^3$)

$$T_{s,sphere} = T_\infty + \frac{\dot{e}_{gen} r_0}{3h}$$

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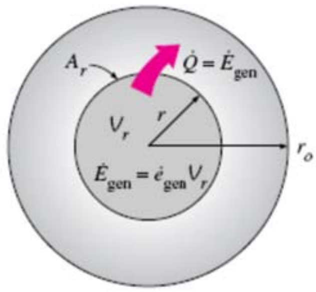
Heat Generation in Solids -The maximum Temperature in a Cylinder (the Centerline)

- The *heat generated* within an inner cylinder must be equal to the *heat conducted* through its outer surface.



$$-kA_r \frac{dT}{dr} = \dot{e}_{gen} V_r$$

- Substituting these expressions into the above equation and separating the variables, we get

$$-k\left(2\pi r L\right)\frac{dT}{dr} = \dot{e}_{gen}\left(\pi r^2 L\right) \rightarrow dT = -\frac{\dot{e}_{gen}}{2k} r dr$$

- Integrating from r=0 where $T(0) = T_0$ to $r=r_o$

$$\Delta T_{\max,cylinder} = T_0 - T_s = \frac{\dot{e}_{gen} r_0^2}{4k}$$

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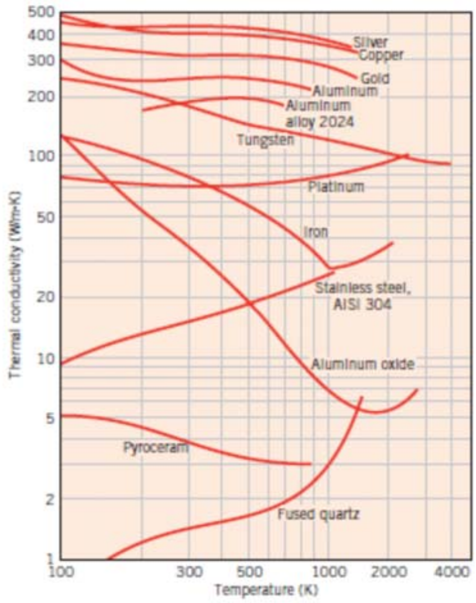
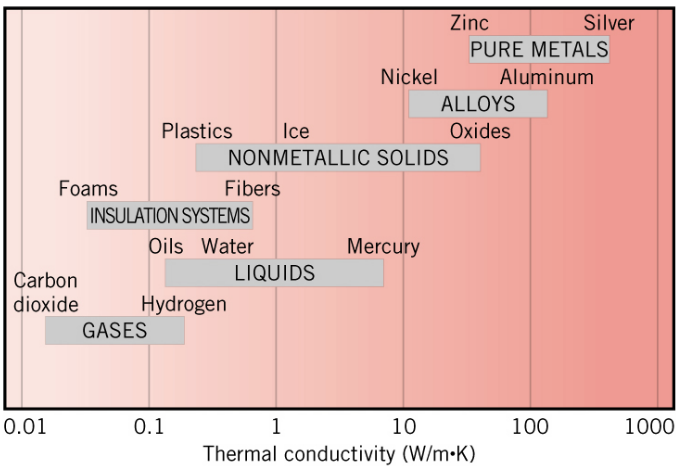
Variable Thermal Conductivity, *k*(*T*)

- The thermal conductivity of a material, in general, varies with temperature.
- An average value for the thermal conductivity is commonly used when the variation is mild.
- This is also common practice for other temperature-dependent properties such as the density and specific heat.



Thermophysical Properties

- Thermal Conductivity: A measure of a material's ability to transfer thermal energy by conduction.



- Thermal Diffusivity: A measure of a material's ability to respond to changes in its thermal environment.

Variable Thermal Conductivity for 1-D Cases

- When the variation of thermal conductivity with temperature $k(T)$ is known, the average value of the thermal conductivity in the temperature range between T_1 and T_2 can be determined from

$$k_{ave} = \frac{\int_{T_1}^{T_2} k(T)dT}{T_2 - T_1}$$

- The variation in thermal conductivity of a material with can often be approximated as a linear function and expressed as

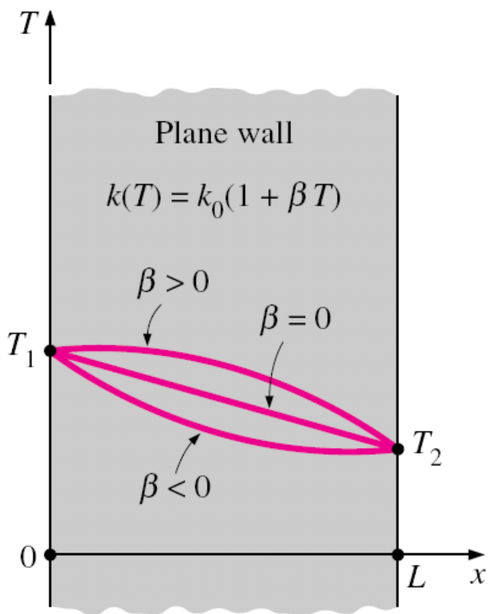
$$k(T) = k_0(1 + \beta T)$$

$$k_{avg} = \frac{\int_{T_1}^{T_2} k_o(1+\beta T)dT}{T_2 - T_1} = k_o\left(1 + \beta\frac{T_2 + T_1}{2}\right) = k(T_{avg})$$

Variable Thermal Conductivity

- For a plane wall the temperature varies linearly during steady one-dimensional heat conduction when the thermal conductivity is constant.

- This is no longer the case when the thermal conductivity changes with temperature (even linearly).



Methodology of a Conduction Analysis

- Consider possible micro- or nanoscale effects in problems involving very small physical dimensions or very rapid changes in heat or cooling rates.

- Solve appropriate form of heat equation to obtain the temperature distribution.

- Knowing the temperature distribution, apply Fourier's Law to obtain the heat flux at any time, location and direction of interest.

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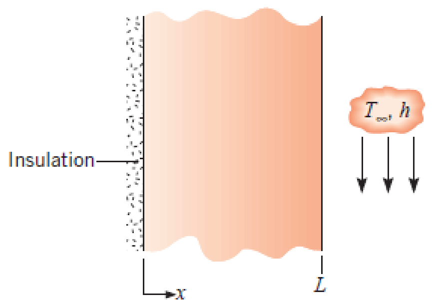
Example

2.57 The plane wall with constant properties and no internal heat generation shown in the figure is initially at a uniform temperature T_i. Suddenly the surface at $x = L$ is heated by a fluid at T_∞ having a convection heat transfer coefficient h. The boundary at $x = 0$ is perfectly insulated.



(a) Write the differential equation, and identify the boundary and initial conditions that could be used to determine the temperature as a function of position and time in the wall.

(b) On $T - x$ coordinates, sketch the temperature distributions for the following conditions: initial condition ($t \leq 0$), steady-state condition ($t \rightarrow \infty$), and two intermediate times.

(c) On $q''_x - t$ coordinates, sketch the heat flux at the locations $x = 0$, $x = L$. That is, show qualitatively how $q''_x(0, t)$ and $q''_x(L, t)$ vary with time.

(d) Write an expression for the total energy transferred to the wall per unit volume of the wall (J/m^3).

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Example cont.

Example cont.

Example cont.

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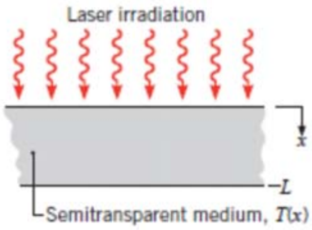
Example cont.

Example

2.37 The steady-state temperature distribution in a semitransparent material of thermal conductivity k and thickness L exposed to laser irradiation is of the form

$$T(x) = -\frac{A}{ka^2}e^{-ax} + Bx + C$$

where A, a, B, and C are known constants. For this situation, radiation absorption in the material is manifested by a distributed heat generation term, $\dot{q}(x)$.



(a) Obtain expressions for the conduction heat fluxes at the front and rear surfaces.

(b) Derive an expression for $\dot{q}(x)$.

(c) Derive an expression for the rate at which radiation is absorbed in the entire material, per unit surface area. Express your result in terms of the known constants for the temperature distribution, the thermal conductivity of the material, and its thickness.

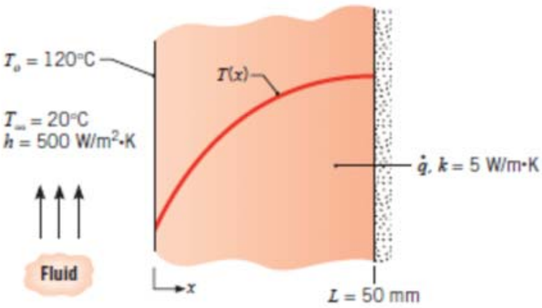
Example cont.

Example cont.

Example cont.

Example

2.34 One-dimensional, steady-state conduction with uniform internal energy generation occurs in a plane wall with a thickness of 50 mm and a constant thermal conductivity of 5 W/m·K. For these conditions, the temperature distribution has the form $T(x) = a + bx + cx^2$. The surface at $x = 0$ has a temperature of $T(0) \equiv T_o = 120°C$ and experiences convection with a fluid for which $T_\infty = 20°C$ and $h = 500\ W/m^2 \cdot K$. The surface at $x = L$ is well insulated.



(a) Applying an overall energy balance to the wall, calculate the volumetric energy generation rate $\dot{q}$.

(b) Determine the coefficients a, b, and c by applying the boundary conditions to the prescribed temperature distribution. Use the results to calculate and plot the temperature distribution.

(c) Consider conditions for which the convection coefficient is halved, but the volumetric energy generation rate remains unchanged. Determine the new values of a, b, and c, and use the results to plot the temperature distribution. *Hint:* recognize that $T(0)$ is no longer 120°C.

(d) Under conditions for which the volumetric energy generation rate is doubled, and the convection coefficient remains unchanged ($h = 500\ W/m^2 \cdot K$), determine the new values of a, b, and c and plot the corresponding temperature distribution. Referring to the results of parts (b), (c), and (d) as Cases 1, 2, and 3, respectively, compare the temperature distributions for the three cases and discuss the effects of h and $\dot{q}$ on the distributions.

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Example cont.

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Example cont.

Example cont.