Using Energy Conservation method to solve spring-mass-damper problem

The total energy in the system has to be constant. This includes:
- Energy dissipated in damper
- Energy introduced by the input force \( f(t) \)
- Energy stored in the spring (potential)
- " " in the mass (kinetic energy)

The energy stored in the spring is
\[
PE = \frac{1}{2} k_s x(t)^2
\]

The energy stored in the mass:
\[
KE = \frac{1}{2} m \dot{x(t)}^2
\]

The energy dissipated by the damper:
\[
\int k_d \dot{x}(t) dx
\]

The energy introduced by the force:
\[
\int f(t) dx
\]
\[ E_{\text{total}} \text{ (original energy in the system)} \]

\[ C + E_{\text{kin}} = KE + PE \]

\[ x(t) \]

\[ C + \int_0^{x(t)} f(x) \, dx - \int_0^{x(t)} k(x) \, dx = \frac{1}{2} m \dot{x}^2(t) \]

\[ + \frac{1}{2} k_s x^2(t) \]

Taking derivatives of both sides with respect to \( t \):

\[ f(t) \cdot \ddot{x}(t) - k_s \dot{x}^2(t) = m \dot{x}(t) \cdot \dot{x}(t) + k_s x(t) \cdot \dot{x}(t) \]

Dividing throughout by \( x(t) \) gives:

\[ f(t) - k_s \dot{x}(t) = m \dot{x}(t) + k_s x(t) \]

\[ f(t) = m \dot{x}(t) + k_s \dot{x}(t) + k_s x(t) \]