Discrete Random Variables

- Definition of Random Variable
- Discrete Random Variable
- Probability Distribution

Def.: Random var. is a variable that assumes numerical values

Random Var.

Discrete | Continuous

* Discrete R.V. can assume a countable # of values

- # of smokers of a sample of size $n=10$
  $X = 0, 1, \ldots, 10$
- # of accidents on a highway
  $X = 0, 1, 2, \ldots$
- # of patients entering the emergency room during 8–10 p.m.

* Continuous R.V. can assume any value over an interval

- Weights of newborn babies
- grades of Math. students
- blood pressure
- temperature
- Time needed to finish a task

Probability Distribution

Example 1: In experiment of tossing 2 coins, let

X: # of heads $= 0, 1, 2$

$P(X=0) = P(\{TT\}) = 1/4$

$P(X=1) = P(\{HT, TH\}) = \frac{2}{4} = \frac{1}{2}$

$P(X=2) = P(\{HH\}) = \frac{1}{4}$

X	0	1	2
$P(X=x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$\leftarrow$ Prob. Dist.

Example 2: X: # of points appear on the face of a dice when the dice is tossed once.

$X = 1, 2, 3, 4, 5, 6$

X	$P(X=x)$
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6
	1

$P(X \geq 4) = \frac{3}{6} = \frac{1}{2}$

$P(1 < X \leq 4) = P(2) + P(3) + P(4)$

$= \frac{3}{6} = \frac{1}{2}$

$P(X \geq 7) = 0$

Expectation

- $EX = \mu = \sum x\, P(x)$

  $P(x)$: Prob. of $X = x$

  (i) $P(x) \geq 0$

  (ii) $\sum_x P(x) = 1$

- Variance $= \sigma^2 = E(X-\mu)^2$

  $= EX^2 - \mu^2$

  $\sigma = \sqrt{EX^2 - \mu^2}$

Example 3 (Go To Example 1). Find EX and σ^2

X	$P(x)$	$xP(x)$	$x^2P(x)$
0	1/4	0	0
1	1/2	1/2	1/2
2	1/4	1/2	1
	1	1	1.5

$EX = \mu = 1$

$EX^2 = 1.5$

$\sigma^2 = EX^2 - \mu^2 = 1.5 - (1)^2$

$= 0.5$

$\sigma = \sqrt{0.5} = \boxed{0.71}$

Example: Given the following distribution

X	1	2	4	10
$P(x)$	0.2	0.4	0.2	0.2

(a) Find $\mu = EX$

(b) Find $\sigma^2 = E(X-\mu)^2$

(c) Find σ

X	$P(x)$	$XP(x)$	$X^2P(x)$
1	0.2	0.2	0.2
2	0.4	0.8	1.6
4	0.2	0.8	3.2
10	0.4	4	40
	1	5.8	45

(a) $\mu = EX = \boxed{5.8}$

(b) $\sigma^2 = EX^2 - \mu^2 = 45 - (5.8)^2 = \boxed{11.36}$

(c) $\sigma = \sqrt{EX^2 - \mu^2} = \boxed{3.37}$

Binomial Random Variable

* We have n identical trials
* Any outcome is either Success (S) or failure (F)
* $p = P(S)$, $q = 1 - p = P(F)$
* Trials are independent
* Our interest is in X: # successes out of n trials

Example: In the experiment of tossing 3 coins,

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

X: # of heads $= 0, 1, 2, 3$

$$P(X=0) = P(TTT) = \left(\tfrac{1}{2}\right)^3 = \binom{3}{0}\left(\tfrac{1}{2}\right)^0\left(\tfrac{1}{2}\right)^3$$

$$P(X=1) = P(HTT) + P(THT) + P(TTH) =$$

$$= \frac{3}{8}$$

$$= \binom{3}{1}\left(\tfrac{1}{2}\right)^1\left(\tfrac{1}{2}\right)^2$$

$$P(X=2) = \frac{3}{8} = \binom{3}{2}\left(\tfrac{1}{2}\right)^2\left(\tfrac{1}{2}\right)$$

$$P(X=3) = \frac{1}{8} = \binom{3}{3}\left(\tfrac{1}{2}\right)^3\left(\tfrac{1}{2}\right)^0$$

In general, $X \sim B(n, p)$

$$P(X=r) = \binom{n}{r} p^r q^{n-r}, \quad r = 0, 1, \ldots, n$$

Binomial Probability is a product of 3 terms

$$P(X=r) = \binom{n}{r} \; p^r \; q^{n-r}$$

$\binom{n}{r}$: # of ways of getting r successes

p^r : Prob. of getting r successes

q^{n-r} : Prob. of getting $(n-r)$ failures

* $\boxed{EX = nP \;,\; \sigma^2 = npq \;,\; \sigma = \sqrt{npq}}$

Example: 40% of students are smoking. A sample of size 10 is taken,

(a) What is the prob. that 2 students are smoking?

$$P(X=2) = \binom{10}{2}(.4)^2(.6)^8 = (45)(.16)(.00066)$$
$$= \boxed{0.0048}$$

(b) What is the prob. that at least one is smoker?

$$P(X \geq 1) = 1 - P(X=0) = 1 - \binom{10}{0}(.4)^0(.6)^{10}$$
$$= 1-(0.6)^{10}$$

(c) Find $EX = \mu$

$$\mu = 10 \times .4 = \boxed{4}$$

(d) Find σ

$$\sigma = \sqrt{10 \times .4 \times .6} = \sqrt{2.4} = \boxed{1.55}$$

Binomial Tables

Table gives cumulative probability $P(X \le r)$

$$T(5) = P(X \le 5) = P(0) + P(1) + \cdots + P(5)$$

$\boxed{n = 10}$

$x \backslash p$	0.01	0.05	.1	.2	.3	.4	.5	– – – –	.99
0						.006			
1						.046			
2						.167			
3						.382			
⋮						.633			
⋮						.834			
7						.945			
8						.988			
						.998			
9						1			
10						1			

$X \sim B(10, .4)$

* $P(X \le 3) = .382$
* $P(X > 3) = 1 - 0.382 = \boxed{0.618}$
* $P(X = 3) = T(3) - T(2) = .382 - .167 = \boxed{0.215}$
* $P(2 < X \le 5) = P(3) + P(4) + P(5)$

$$= [0.215] + [.633 - .382] + [.834 - .633]$$

$$= .215 + .251 + .201$$

$$= \boxed{0.667}$$