Ch.7 (Inference based on a Single Sample)

- Estimate a parameter based on a large sample
- Use a confidence interval for estimating the parameter
- Determination the sample size needed to estimate the parameter.

Large Sample Confidence Interval for a Population Mean

Let μ : T.V. Watch Average !!!

A sample of 64 persons watching T.V. shows that

$\bar{X} = 30$ hours/week

$S = 10$ hours

Can you estimate the watch average of all persons in the population?

* A point estimate for the total average μ is

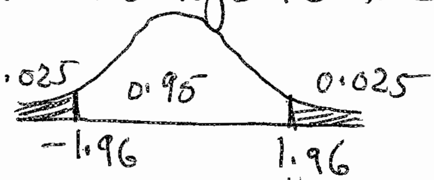
$\bar{X} = 30$ hours/week.

* To find confidence interval (C.I.) for μ, we assume the normal dist. for T.V. hours.

$\longleftarrow$ $\bar{X} = 30$ hrs. $\longrightarrow$

The limits of C.I. depend on the following: ②

(1) The sampling distribution of the point estimator of μ [point estimator of $\mu = \bar{X}$]. If we are confident 95% that μ belongs to the C.I., then from normal dist.



$$= P(Z \le 1.96) = 0.975$$

(2) Standard deviation of the estimator $\bar{X}$ = standard error of $\bar{X} = \frac{\sigma}{\sqrt{n}}$

$\Rightarrow$ 95% C.I. for μ is $\bar{X} \pm 1.96 \frac{\sigma}{\sqrt{n}}$

$$30 \pm 1.96 \frac{10}{\sqrt{64}}$$

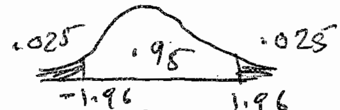
$$30 \pm 1.96\,(1.25)$$

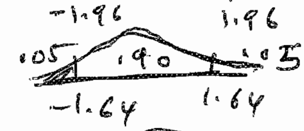
$$30 \pm 2.45 \Longrightarrow \underline{(27.55, 32.45)}$$

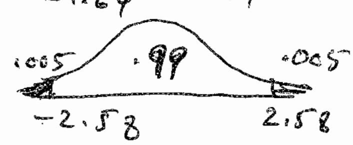
In general, C.I. for μ is $\bar{X} \pm z \frac{\sigma}{\sqrt{n}}$

z is based on confidence level

Confidence level	C.I. for μ
95%	$\bar{X} \pm 1.96 \frac{\sigma}{\sqrt{n}}$
90%	$\bar{X} \pm 1.64 \frac{\sigma}{\sqrt{n}}$
99%	$\bar{X} \pm 2.58 \frac{\sigma}{\sqrt{n}}$
$(1-\alpha)\%$ $[0<\alpha<1]$	$\bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$







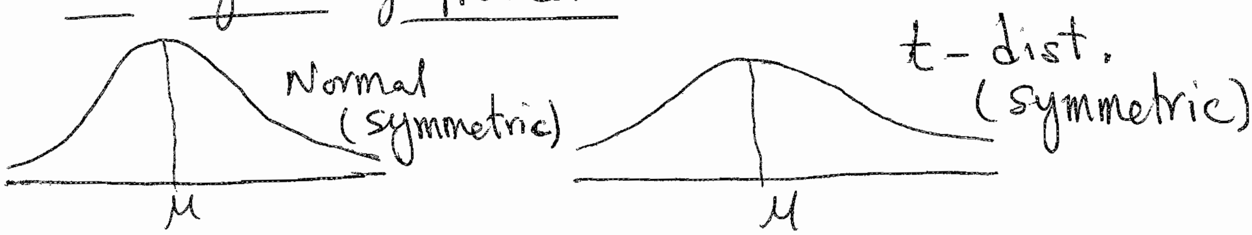
Note: If σ is not known, then we replace σ by s since n is large

C.I. for μ: $\bar{X} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$

Small Sample C.I. for μ

Set-up: (1) Unknown σ (2) small sample

The sampling distribution of $\bar{X}$ is Student's distribution with $n-1$ degrees of freedom

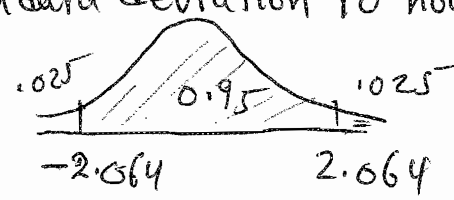


As $n \to \infty$ (n large), t-dist. $\approx$ Normal dist.

$\Rightarrow$ Small sample ($n<30$), C.I. for μ is

$$\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

Example: Suppose a sample of 25 persons watch an average of 30 hours/week with standard deviation 10 hours. Find 95% C.I. for μ



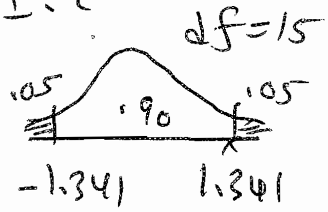
degrees of freedom $= df = n-1 = 24$

$$\bar{X} \pm 2.064 \frac{s}{\sqrt{n}} = 30 \pm 2.064\left(\frac{10}{\sqrt{25}}\right)$$

$$= 30 \pm 4.13$$

$$= (25.87, 34.13)$$

<u>Example</u>: We wish to estimate the average of the Cholestrol levels of the patients in Sabah Hospital. A sample of size 16 shows an average $\bar{X} = 240$ and standard deviation 12. Estimate of the average of the cholestrol levels μ by 90 % C.I.?



$n = 16 \Rightarrow df = 15, \; t_{\alpha/2} = 1.341$

$240 \pm 1.341\left(\frac{12}{\sqrt{16}}\right)$

$240 \pm 1.341(3) \Longrightarrow 240 \pm 4.023$

$\Longrightarrow (245.977, 244.023)$

<u>C.I. for a Population Proportion</u>

- p: Population Proportion

  $\hat{p}$ = Sample Proportion $= \frac{X}{n} = \frac{\#\text{ of Successes out of } n}{n}$

- The mean of $\hat{p} = \mu_{\hat{p}} = p$

  The standard deviation of $\hat{p}$ is $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$, $q = 1 - p$

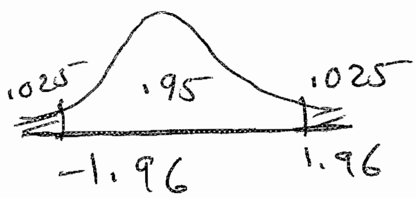
- C.I. for p is (for large n)

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Example: A researcher would like to estimate the proportion of people who are choosing an islamic candidate in the election. A sample of 100 persons shows that 60 persons are choosing this candidate.

(a) Find 95% C.I. for p = prop. of people choosing an islamic candidate

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$



$$\hat{p} = \frac{60}{100} = 0.6$$

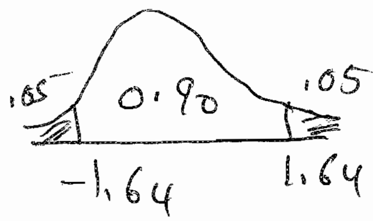
$$0.6 \pm 1.96\sqrt{\frac{(0.6)(0.4)}{100}} \quad \text{or} \quad 0.6 \pm 1.96(0.05)$$

$$0.6 \pm 0.098$$

$$\Downarrow$$

$$(0.50, 0.7)$$

(b) Find 90% C.I. for p



$$z_{\alpha/2} = 1.64$$

$$0.6 \pm 1.64\sqrt{\frac{(0.6)(0.4)}{100}}$$

$$0.6 \pm (1.64)(0.049) \Rightarrow 0.6 \pm 0.08$$

$$\Rightarrow (0.52, 0.68)$$

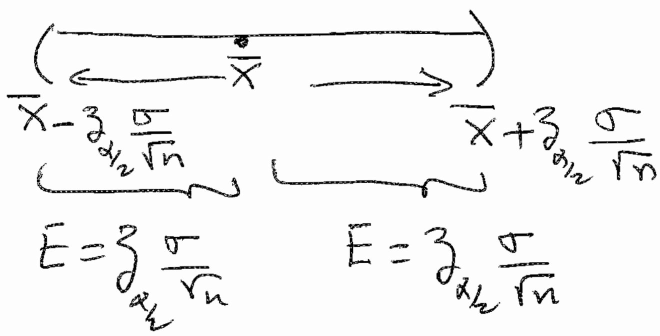
Determination the Sample Size

- For estimating μ

  E = error of estimation

  $$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

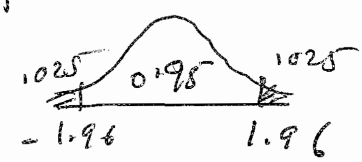
  Width of C.I. $= 2 \cdot E$



- The sample size needed to estimate μ by C.I.

$$n = \frac{(z_{\alpha/2})^2\sigma^2}{E^2} \quad \text{or} \quad n = \left(\frac{z_{\alpha/2}}{E}\right)^2\sigma^2$$

Example: One wish to estimate the age average of the patients in a certain hospital within 3 years with 95% C.I. If the standard deviation is 10, find the # of patients needed to make the estimate.



$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2\sigma^2$$

$$= \left(\frac{1.96}{3}\right)^2 100 = 42.68 \approx 43$$

- For estimating p,

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 pq$$

Either p is estimated from previous studies call it p^*

or $p = 0.5$ if no previous studies.

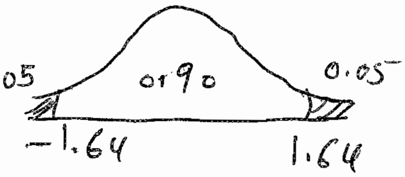
<u>Example</u>: How many cell phones must a manufacturer test to estimate the proportion of defective (P), to within 0.01 with 90% C.I. if

(a) an initial estimate of p is 0.10

(b) no initial estimate of p

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p q$$

(a) $n = \left(\frac{1.64}{0.01}\right)^2 (0.1)(0.9)$



$= (26896)(0.1)(0.9)$

$= 2420.64 \approx \boxed{2421}$

(b) $n = \left(\frac{1.64}{0.01}\right)^2 (0.5)(0.5)$

$= (26896)(0.5)(0.5)$

$= \boxed{6724}$