Chapter 8 (Test of Hypotheses)

Up to now, we have studied

- Point estimators of population parameters
- Confidence Intervals for population parameters

Need

Test a specific value of a pop. parameter

8.1 Elements of the Hypotheses Testing

* **Types of hypotheses**

(1) Null hypothesis H_0

(2) Alternative hypothesis H_1

* **Test Statistic** : Measure computed based on the sample to help us in taking a decision

* **Decision** → Reject H_0
→ Accept H_0

Decision	States of Nature: H_0	H_1
Reject H_0	Type I Error	✓
Accept H_0	✓	Type II Error

$\alpha = P(\text{Type I error}) = P(\text{reject } H_0 \mid H_0 \text{ is true})$

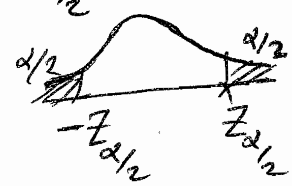
α is called significance level مستوى الدلالة الاحصائية

8.2 Large-Sample Test about μ

$H_0: \mu = \mu_0$ vs. $H_1: \mu > \mu_0$, Reject H_0 if $Z > Z_\alpha$

$< \mu_0$, Reject H_0 if $Z < -Z_\alpha$

$\neq \mu_0$, Reject H_0 if $Z > Z_{\alpha/2}$ or $Z < -Z_{\alpha/2}$



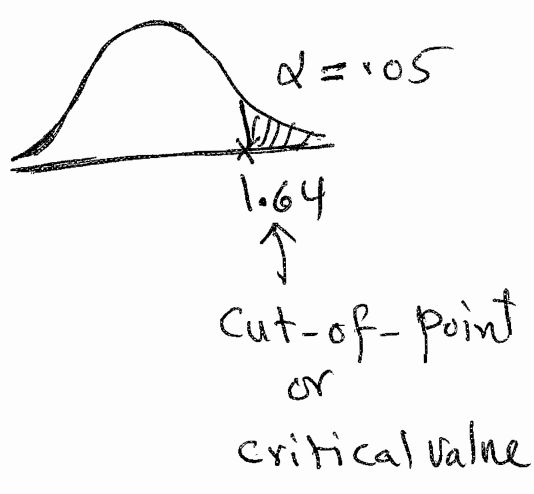
$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

Example 1: The salaries of male employees are normally distributed. A sample of 64 employees provided that their salary average is 2400 KD/month and standard deviation 400 KD/month. We wish to test at $\alpha = 0.05$ level, $H_0: \mu = 2200$ vs. $H_1: \mu > 2200$.

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{2400 - 2200}{400/8} = \frac{1600}{400} = 4$$

Since observed $Z = 4 > 1.64$,

We reject H_0.



Example 2: The grades of Exam 1 of STAT 101 are normally distributed. A sample of 100 students is taken and showed that its mean 75 and standard deviation 20. Do the above data support the claim that the mean of grades is different from 72? ~~Use~~

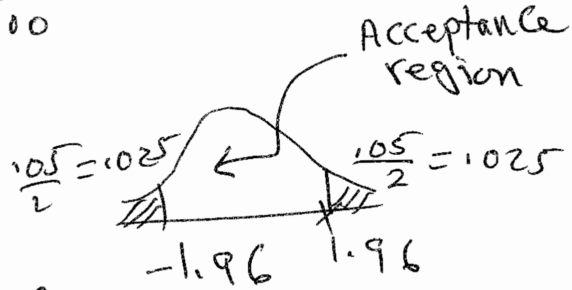
(i) Use $\alpha = 0.05$

(ii) Use $\alpha = 0.10$

Solution: $H_0: \mu = 72$ vs. $H_1: \mu \neq 72$

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{75-72}{20/\sqrt{100}} = \boxed{1.5}$$

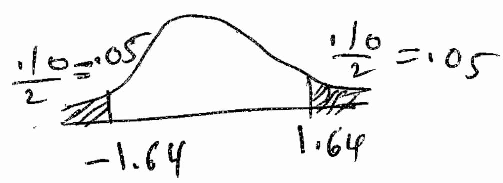
(i) $\alpha = 0.05$



observed value $= 1.5 \in$ acceptance region

$\Rightarrow$ Accept H_0

(ii) $\alpha = 0.10$



observed value $Z = 1.5 \in$ acceptane region

$\Rightarrow$ Accept H_0

Observed Significance Level [P-value]

Suppose $Z = 2.12$. $P(Z > 2.12) = 0.017$

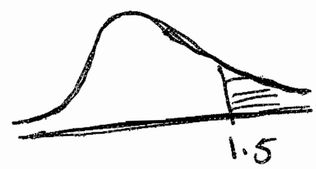
Reject H_0 at $\alpha = 0.05$ level

Accept H_0 at $\alpha = 0.01$ level

Back to Example 2

* $H_0: \mu = 72$ vs. $H_1: \mu > 72$

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{75 - 72}{20 / \sqrt{100}} = \boxed{1.5}$$



p-value $= P(Z > 1.5)$ = Area to the right of 1.5

$= 1 - 0.9332 = \boxed{0.1668}$

Test Results: If p-value $< \alpha \Rightarrow$ reject H_0
If p-value $> \alpha \Rightarrow$ accept H_0

For $\alpha = 0.05$, p-value $= 0.1668 > \alpha = 0.05$
$\Rightarrow$ Accept H_0 at 0.05 level

* $H_0: \mu = 72$ Vs. $H_1: \mu \neq 72$

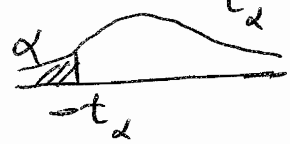
p-value $= 2\,P(Z > 1.5) = 2(0.1668) = \boxed{0.3336}$

Small Sample Test about μ

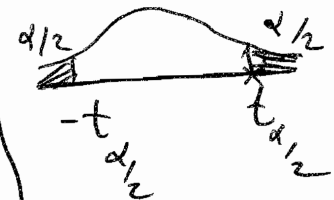
In this case, we use t-test $\quad t = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$

$H_0: \mu = \mu_0$ vs. $H_1: \mu > \mu_0$, reject H_0 if $\boxed{t > t_\alpha}$

$< \mu_0$, " " " $\boxed{t < -t_\alpha}$



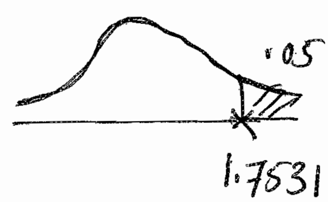
$\neq \mu_0$, " " " $\boxed{t > t_{\alpha/2} \text{ or } t < -t_{\alpha/2}}$



Example 3: The grades of Exam I are normally distributed. A sample of 16 grades is taken and provided $\bar{X} = 75$ and $s = 10$. Can you conclude that the grade average is exceeding 72? Use $\alpha = 0.05$

$H_0: \mu = 72$ vs. $H_1: \mu > 72$

$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{75 - 72}{10/\sqrt{16}} = \boxed{1.2}$$



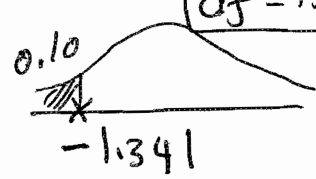
$\not> 1.7531 \leftarrow$ critical value

$\Rightarrow$ accept H_0

– Can you say that the true mean is less than 79? $\boxed{\alpha = 0.10}$ ~~α=0.01~~

$H_0: \mu = 79$ vs. $H_1: \mu < 77$ $\quad$ (df = 15)

$$t = \frac{75 - 79}{10/\sqrt{16}} = \frac{-16}{10} = \boxed{-1.6}$$



Since observed $t = -1.6 <$ critical value $= -1.341$

we reject H_0

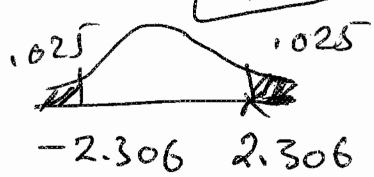
Example 4 : The weights of new born babies are normally distributed. A sample of $n = 9$ babies provided an average 3.5 Kg and standard deviation 2.5 Kg.

Test $H_0 : \mu = 3$ vs. $H_1 : \mu \neq 3$ at $\alpha = 0.05$

$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{3.5 - 3}{2.5/\sqrt{9}} = \frac{1.5}{2.5} = \frac{3}{5} = \boxed{0.6}$$

$\boxed{df = 8}$

Since observed t belongs to acceptance region,



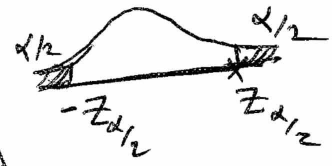
we accept H_0

Large-Sample Test for P

$H_0: P = P_0$ vs. $H_1: P > P_0$, reject H_0 if $\boxed{Z > Z_\alpha}$

$< P_0$, reject H_0 if $\boxed{Z < -Z_\alpha}$

$\neq P_0$, reject H_0 if $\boxed{Z > Z_{\alpha/2} \text{ or } Z < -Z_{\alpha/2}}$



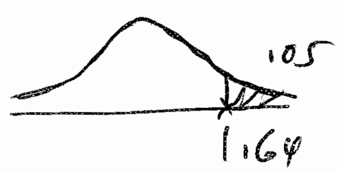
$$Z = \frac{\hat{P} - P_0}{\sqrt{\frac{P_0 q_0}{n}}}$$

<u>Condition:</u> Large n ($n \geq 30$)

<u>Example 5</u>: A sample of 100 persons is taken and it is found that 60 of them are smokers. Test at $\alpha = 0.05$.

(a) $H_0: P = 0.5$ vs. $H_1: P > 0.5$

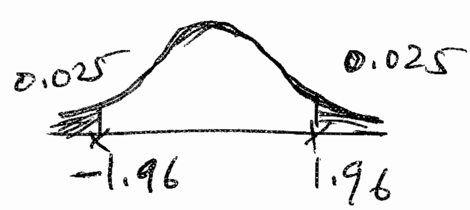
$$Z = \frac{0.6 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{100}}} = 2 > 1.64$$



$\Rightarrow$ Rej. H_0

(b) ~~Test~~ $H_0: P = 0.5$ vs. $H_1: P \neq 0.5$

$$Z = \frac{0.6 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{100}}} = 2 > 1.96$$



$\Rightarrow$ Rej. H_0