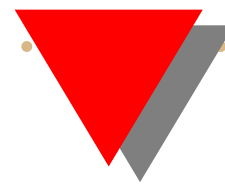


Bayes Theorem

- (1) Partitioning the sample space into B_1, B_2, \dots, B_k disjoint events ($P(B_i \cap B_j) = 0$ for $i \neq j$).
- (2) Given $P(A|B_1), \dots, P(A|B_k)$ which represent prior probabilities
- (3) Required: $P(B_i|A)$

Byes Theorem:

$$\begin{aligned} P(B_i|A) &= \frac{P(B_i \cap A)}{P(A)} \\ &= \frac{P(A|B_i)P(B_i)}{\sum_{i=1}^k P(A \cap B_i)} \\ &= \frac{P(A|B_i)P(B_i)}{\sum_{i=1}^k P(A|B_i)P(B_i)} \end{aligned}$$



Example: The mails in US are delivered by three companies FAD, CAD and NAD with

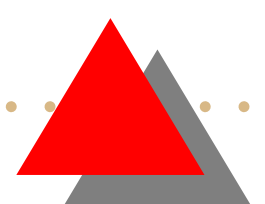
$$P(FAD) = 0.4, P(CAD) = 0.3, P(NAD) = 0.3$$

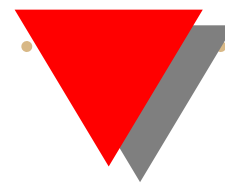
Also

$$P(Late|FAD) = 0.2, P(Late|CAD) = 0.4, P(Late|NAD) = 0.5$$

(1) Given the mail is late, what is the probability that it was delivered by FAD.

$$\begin{aligned} P(FAD|L) &= \frac{0.08}{0.08 + 0.12 + 0.15} \\ &= \frac{0.08}{0.35} \\ &= 0.23 \end{aligned}$$





Cont./ Example:

(2) Given the mail is late, what is the probability that it was delivered by FAD or NAD.

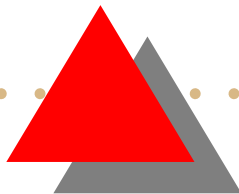
$$\begin{aligned} P(FAD|L) + P(NAD|L) &= \frac{0.08 + 0.15}{0.08 + 0.12 + 0.15} \\ &= \frac{0.23}{0.35} \\ &= 0.66 \end{aligned}$$

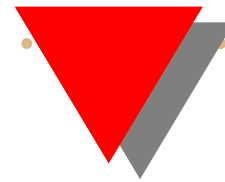
(3) What is the probability that it was late?

$$P(Late) = 0.08 + 0.12 + 0.15 = 0.35$$

(4) Given the mail is not late, What is the probability that it was delivered by CAD?

$$P(CAD|\bar{L}) = \frac{0.18}{0.65} = 0.28$$





Example:

Box I contains 3 Red and 2 Black balls

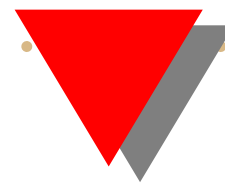
Box II contains 1 Red and 4 Black balls

If 2 balls are transferred from Box I into Box II and then one ball is drawn from Box II. If the ball drawn from Box II was red, what is the probability the balls transferred from Box I are both red.

Solution: Here one can use the tree argument (what is that?).

$$\begin{aligned} P(\text{Balls transferred from Box I are red} | \text{Ball drawn from II was red}) &= \frac{9/70}{(9 + 12 + 1)/70} \\ &= \frac{9}{22} \end{aligned}$$





Example: If

$$P(A \cap \bar{B}) = 0.4, \quad P(\bar{A} \cap B) = 0.2, \quad \text{and } P(\bar{A} \cap \bar{B}) = 0.3.$$

Find

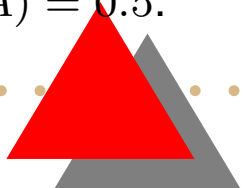
- (i) $P(A)$
- (ii) $P(\bar{A})$
- (iii) $P(\bar{A}|B)$


Solution: From $P(\bar{A} \cap \bar{B}) = 0.3$, we have $P(A \cup B) = 0.7$. Now

(i) $P(A \cap B) = 0.1$ and then

$$\begin{aligned} P(A \cap \bar{B}) &= P(A) - P(A \cap B) \\ 0.4 &= P(A) - 0.1 \end{aligned}$$

Then $P(A) = 0.5$.





Cont./Example: (ii) $P(A \cap B) = 0.1$ and then

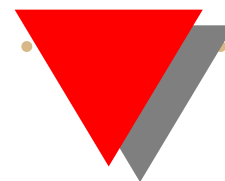
$$\begin{aligned}P(\bar{A} \cap B) &= P(B) - P(A \cap B) \\0.2 &= P(B) - 0.1\end{aligned}$$

Then $P(A) = 0.3$.

(ii) $P(\bar{A}|B)$

$$\begin{aligned}P(\bar{A}|B) &= \frac{P(\bar{A} \cap B)}{P(B)} \\&= \frac{0.2}{0.3} \\&= \frac{2}{3}\end{aligned}$$





Example:

Out of 12 people applying for a job, 3 cannot do the work. Suppose that 2 persons will be hired.

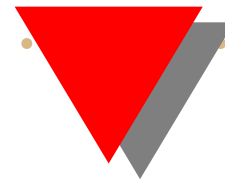
- (a) In how many ways will 0 or 1 people not be able to do the work
- (b) What is the probability that 0 or 1 people not be able to do the work
- (c) If 2 persons are chosen in random, what is the probability that neither will be able to do the job.

Solution:

- (a) In how many ways will 0 or 1 people not be able to do the work

$$\begin{aligned} &= \binom{3}{0} \binom{9}{2} + \binom{3}{1} \binom{9}{1} \\ &= \frac{(9)(8)}{(1)(2)} + (3)(9) \\ &= 36 + 27 \\ &= 63 \end{aligned}$$



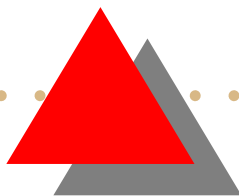


Cont./ Example: (b) What is the probability that 0 or 1 people not be able to do the work

$$\begin{aligned} &= P(X = 0) + P(X = 1) \\ &= \frac{\binom{3}{0} \binom{9}{2} + \binom{3}{1} \binom{9}{1}}{\binom{12}{2}} \\ &= \frac{63}{66} \\ &= \frac{21}{22} \end{aligned}$$

(c) If 2 persons are chosen in random, what is the probability that neither will be able to do the job.

$$\begin{aligned} &= \frac{\binom{3}{2} \binom{9}{0}}{\binom{12}{2}} \\ &= \frac{1}{22} \end{aligned}$$





Example:

For each of 20 questions on a multiple-choice test, a student can choose one of 5 answers

- (a) How many different sets of answers are possible?
- (b) If a person guessed on all the questions, what is the probability that all of the questions would be answered correctly?

Answer:

- (a) How many different sets of answers are possible?

$$\text{answer} = 5^{20}$$

- (b) If a person guessed on all the questions, what is the probability that all of the questions would be answered correctly?

$$\text{Probability} = \frac{1}{5^{20}}$$

