

Table A.5. Upper tail probabilities for the null distribution of Wilcoxon's rank sum W statistic: $m = 3(1)10, n = 1(1)m; m = 11(1)20, n = 1(1)4$

For given m and n , the table entry for the point x is $P_0\{W \geq x\}$. Under these conditions, if x is such that $P_0\{W \geq x\} = \alpha$, then $w(\alpha, m, n) = x$.

$n = 1$									
x	$m = 3$	$m = 4$	$m = 5$	$m = 6$	$m = 7$	$m = 8$	$m = 9$	$m = 10$	$m = 11$
3	.500	.600							
4	.250	.400	.500	.571					
5		.200	.333	.429	.500	.556			
6			.167	.286	.375	.444	.500	.545	
7				.143	.250	.333	.400	.455	.500
8					.125	.222	.300	.364	.417
9						.111	.200	.273	.333
10							.100	.182	.250
11								.091	.167
12									.083

$n = 1$									
x	$m = 12$	$m = 13$	$m = 14$	$m = 15$	$m = 16$	$m = 17$	$m = 18$	$m = 19$	$m = 20$
7	.538								
8	.462	.500	.533						
9	.385	.429	.467	.500	.529				
10	.308	.357	.400	.438	.471	.500	.526		
11	.231	.286	.333	.375	.412	.444	.474	.500	.524
12	.154	.214	.267	.312	.353	.389	.421	.450	.476
13	.077	.143	.200	.250	.294	.333	.368	.400	.429
14		.071	.133	.188	.235	.278	.316	.350	.381
15			.067	.125	.176	.222	.263	.300	.333
16				.062	.118	.167	.211	.250	.286
17					.059	.111	.158	.200	.238
18						.056	.105	.150	.190
19							.053	.100	.143
20								.050	.095
21									.048

Table A.5 (continued)

$n = 2$									
x	$m = 3$	$m = 4$	$m = 5$	$m = 6$	$m = 7$	$m = 8$	$m = 9$	$m = 10$	$m = 11$
6	.600								
7	.400	.600							
8	.200	.400	.571						
9	.100	.267	.429	.571					
10		.133	.286	.429	.556				
11		.067	.190	.321	.444	.556			
12			.095	.214	.333	.444	.545		
13			.048	.143	.250	.356	.455	.545	
14				.071	.167	.267	.364	.455	.538
15				.036	.111	.200	.291	.379	.462
16					.056	.133	.218	.303	.385
17					.028	.089	.164	.242	.321
18						.044	.109	.182	.256
19						.022	.073	.136	.205
20							.036	.091	.154
21							.018	.061	.115
22								.030	.077
23								.015	.051
24									.026
25									.013

$n = 2$									
x	$m = 12$	$m = 13$	$m = 14$	$m = 15$	$m = 16$	$m = 17$	$m = 18$	$m = 19$	$m = 20$
15	.538								
16	.462	.533							
17	.396	.467	.533						
18	.330	.400	.467	.529					
19	.275	.343	.408	.471	.529				
20	.220	.286	.350	.412	.471	.526			
21	.176	.238	.300	.360	.418	.474	.526		
22	.132	.190	.250	.309	.366	.421	.474	.524	
23	.099	.152	.208	.265	.320	.374	.426	.476	.524
24	.066	.114	.167	.221	.275	.327	.379	.429	.476
25	.044	.086	.133	.184	.235	.287	.337	.386	.433
26	.022	.057	.100	.147	.196	.246	.295	.343	.390
27	.011	.038	.075	.118	.163	.211	.258	.305	.351
28		.019	.050	.088	.131	.175	.221	.267	.312
29		.010	.033	.066	.105	.146	.189	.233	.277
30			.017	.044	.078	.117	.158	.200	.242
31			.008	.029	.059	.094	.132	.171	.212
32				.015	.039	.070	.105	.143	.182

Table A.5 (continued)

 $n = 2$

x	$m = 12$	$m = 13$	$m = 14$	$m = 15$	$m = 16$	$m = 17$	$m = 18$	$m = 19$	$m = 20$
33				.007	.026	.053	.084	.119	.156
34					.013	.035	.063	.095	.130
35					.007	.023	.047	.076	.108
36						.012	.032	.057	.087
37						.006	.021	.043	.069
38							.011	.029	.052
39							.005	.019	.039
40								.010	.026
41								.005	.017
42									.009
43									.004

 $n = 3$

x	$m = 3$	$m = 4$	$m = 5$	$m = 6$	$m = 7$	$m = 8$	$m = 9$	$m = 10$	$m = 11$
11	.500								
12	.350	.571							
13	.200	.429							
14	.100	.314	.500						
15	.050	.200	.393	.548					
16		.114	.286	.452					
17		.057	.196	.357	.500				
18		.029	.125	.274	.417	.539			
19			.071	.190	.333	.461			
20			.036	.131	.258	.388	.500		
21			.018	.083	.192	.315	.432	.531	
22				.048	.133	.248	.364	.469	
23				.024	.092	.188	.300	.406	.500
24				.012	.058	.139	.241	.346	.442
25					.033	.097	.186	.287	.385
26					.017	.067	.141	.234	.330
27					.008	.042	.105	.185	.277
28						.024	.073	.143	.228
29						.012	.050	.108	.184
30						.006	.032	.080	.146
31							.018	.056	.113
32							.009	.038	.085
33							.005	.024	.063
34								.014	.044
35								.007	.030
36								.003	.019
37									.011
38									.005
39									.003

Table A.5 (continued)

 $n = 3$

x	$m = 12$	$m = 13$	$m = 14$	$m = 15$	$m = 16$	$m = 17$	$m = 18$	$m = 19$	$m = 20$
24	.527								
25	.473								
26	.420	.500							
27	.367	.450	.524						
28	.316	.400	.476						
29	.268	.352	.429	.500					
30	.224	.305	.384	.456	.521				
31	.182	.261	.338	.412	.479				
32	.147	.220	.296	.369	.438	.500			
33	.116	.182	.254	.327	.396	.461	.519		
34	.090	.148	.216	.287	.356	.421	.481		
35	.068	.120	.181	.249	.317	.382	.444	.500	
36	.051	.095	.150	.213	.280	.345	.407	.464	.517
37	.035	.073	.122	.180	.244	.308	.370	.429	.483
38	.024	.055	.099	.151	.211	.273	.335	.394	.449
39	.015	.041	.078	.125	.180	.239	.300	.359	.415
40	.009	.029	.060	.102	.152	.208	.267	.325	.382
41	.004	.020	.046	.082	.127	.179	.235	.293	.349
42	.002	.012	.034	.065	.105	.153	.206	.262	.317
43		.007	.024	.050	.086	.129	.178	.232	.286
44		.004	.016	.038	.069	.108	.153	.204	.257
45		.002	.010	.028	.055	.089	.131	.178	.229
46			.006	.020	.042	.073	.111	.154	.202
47			.003	.013	.032	.059	.092	.132	.177
48			.001	.009	.024	.046	.077	.113	.155
49				.005	.017	.036	.062	.095	.134
50				.002	.011	.027	.050	.080	.115
51				.001	.007	.020	.040	.066	.098
52					.004	.014	.031	.054	.083
53					.002	.010	.023	.044	.069
54					.001	.006	.017	.034	.058
55						.004	.012	.027	.047
56						.002	.008	.020	.038
57						.001	.005	.015	.030
58							.003	.010	.023
59							.002	.007	.018
60							.001	.005	.013
61								.003	.009
62								.001	.006
63								.001	.004
64									.002
65									.001
66									.001

Table A.5 (continued)

n = 4

x	m = 4	m = 5	m = 6	m = 7	m = 8	m = 9	m = 10	m = 11
18	.557							
19	.443							
20	.343	.548						
21	.243	.452						
22	.171	.365	.543					
23	.100	.278	.457					
24	.057	.206	.381	.536				
25	.029	.143	.305	.464				
26	.014	.095	.238	.394	.533			
27		.056	.176	.324	.467			
28		.032	.129	.264	.404	.530		
29		.016	.086	.206	.341	.470		
30		.008	.057	.158	.285	.413	.527	
31			.033	.115	.230	.355	.473	
32			.019	.082	.184	.302	.420	.525
33			.010	.055	.141	.252	.367	.475
34			.005	.036	.107	.207	.318	.426
35				.021	.077	.165	.270	.377
36				.012	.055	.130	.227	.330
37				.006	.036	.099	.187	.286
38				.003	.024	.074	.152	.245
39					.014	.053	.120	.206
40					.008	.038	.094	.171
41					.004	.025	.071	.140
42					.002	.017	.053	.113
43						.010	.038	.089
44						.006	.027	.069
45						.003	.018	.052
46						.001	.012	.039
47							.007	.028
48							.004	.020
49							.002	.013
50							.001	.009
51								.005
52								.003
53								.001
54								.001

Table A.5 (continued)

n = 4

x	m = 12	m = 13	m = 14	m = 15	m = 16	m = 17	m = 18	m = 19	m = 20
34	.524								
35	.476								
36	.431	.522							
37	.385	.478							
38	.342	.435	.521						
39	.299	.392	.479						
40	.260	.352	.439	.519					
41	.223	.312	.399	.481					
42	.190	.274	.360	.443	.518				
43	.158	.239	.323	.405	.482				
44	.131	.206	.287	.368	.446	.517			
45	.106	.175	.253	.332	.410	.483			
46	.085	.148	.221	.298	.375	.449	.516		
47	.066	.123	.191	.265	.341	.415	.484		
48	.052	.101	.164	.235	.308	.381	.451	.516	
49	.039	.082	.139	.205	.277	.349	.419	.484	
50	.029	.065	.116	.179	.247	.318	.387	.453	.515
51	.021	.051	.096	.154	.219	.287	.356	.422	.485
52	.015	.039	.079	.131	.192	.258	.326	.392	.455
53	.010	.030	.063	.110	.168	.231	.297	.363	.426
54	.007	.022	.051	.092	.145	.205	.269	.334	.397
55	.004	.016	.040	.076	.124	.181	.242	.306	.368
56	.002	.011	.031	.062	.106	.158	.217	.279	.341
57	.001	.008	.023	.050	.089	.138	.193	.253	.314
58	.001	.005	.017	.040	.074	.119	.171	.228	.288
59		.003	.012	.031	.061	.101	.150	.205	.262
60		.002	.009	.024	.050	.086	.131	.183	.239
61		.001	.006	.018	.040	.072	.113	.162	.216
62		.000	.004	.014	.032	.060	.098	.143	.194
63			.002	.010	.025	.049	.083	.125	.174
64			.001	.007	.019	.040	.070	.109	.155
65			.001	.005	.015	.032	.059	.094	.137
66			.000	.003	.011	.026	.049	.081	.120
67				.002	.008	.020	.040	.069	.105
68				.001	.006	.016	.033	.058	.091
69				.001	.004	.012	.027	.049	.079
70				.000	.002	.009	.021	.041	.067
71					.001	.006	.017	.033	.057
72					.001	.005	.013	.027	.048
73					.000	.003	.010	.022	.041
74						.002	.007	.018	.034
75						.001	.005	.014	.028
76						.001	.004	.011	.023
77						.000	.002	.008	.018

Table A.5 (continued)

n = 4									
x	m = 12	m = 13	m = 14	m = 15	m = 16	m = 17	m = 18	m = 19	m = 20
78						.000	.002	.006	.015
79							.001	.004	.011
80							.001	.003	.009
81							.000	.002	.007
82							.000	.001	.005
83								.001	.004
84								.000	.003
85								.000	.002
86								.000	.001
87									.001
88									.000
89									.000
90									.000

n = 5						
x	m = 5	m = 6	m = 7	m = 8	m = 9	m = 10
28	.500					
29	.421					
30	.345	.535				
31	.274	.465				
32	.210	.396				
33	.155	.331	.500			
34	.111	.268	.438			
35	.075	.214	.378	.528		
36	.048	.165	.319	.472		
37	.028	.123	.265	.416		
38	.016	.089	.216	.362	.500	
39	.008	.063	.172	.311	.449	
40	.004	.041	.134	.262	.399	.523
41		.026	.101	.218	.350	.477
42		.015	.074	.177	.303	.430
43		.009	.053	.142	.259	.384
44		.004	.037	.111	.219	.339
45		.002	.024	.085	.182	.297
46			.015	.064	.149	.257
47			.009	.047	.120	.220
48			.005	.033	.095	.185
49			.003	.023	.073	.155
50			.001	.015	.056	.127
51				.009	.041	.103
52				.005	.030	.082

Table A.5 (continued)

n = 5						
x	m = 5	m = 6	m = 7	m = 8	m = 9	m = 10
53				.003	.021	.065
54				.002	.014	.050
55				.001	.009	.038
56					.006	.028
57					.003	.020
58					.002	.014
59					.001	.010
60					.000	.006
61						.004
62						.002
63						.001
64						.001
65						.000

n = 6					
x	m = 6	m = 7	m = 8	m = 9	m = 10
39	.531				
40	.469				
41	.409				
42	.350	.527			
43	.294	.473			
44	.242	.418			
45	.197	.365	.525		
46	.155	.314	.475		
47	.120	.267	.426		
48	.090	.223	.377	.523	
49	.066	.183	.331	.477	
50	.047	.147	.286	.432	
51	.032	.117	.245	.388	.521
52	.021	.090	.207	.344	.479
53	.013	.069	.172	.303	.437
54	.008	.051	.141	.264	.396
55	.004	.037	.114	.228	.356
56	.002	.026	.091	.194	.318
57	.001	.017	.071	.164	.281
58		.011	.054	.136	.246
59		.007	.041	.112	.214
60		.004	.030	.091	.184
61		.002	.021	.072	.157
62		.001	.015	.057	.132
63		.001	.010	.044	.110

Table A.5 (continued)

$n = 6$					
x	$m = 6$	$m = 7$	$m = 8$	$m = 9$	$m = 10$
64			.006	.033	.090
65			.004	.025	.074
66			.002	.018	.059
67			.001	.013	.047
68			.001	.009	.036
69			.000	.006	.028
70				.004	.021
71				.002	.016
72				.001	.011
73				.001	.008
74				.000	.005
75				.000	.004
76					.002
77					.001
78					.001
79					.000
80					.000
81					.000

n = 7				
x	m = 7	m = 8	m = 9	m = 10
53	.500			
54	.451			
55	.402			
56	.355	.522		
57	.310	.478		
58	.267	.433		
59	.228	.389		
60	.191	.347	.500	
61	.159	.306	.459	
62	.130	.268	.419	
63	.104	.232	.379	.519
64	.082	.198	.340	.481
65	.064	.168	.303	.443
66	.049	.140	.268	.406
67	.036	.116	.235	.370
68	.027	.095	.204	.335
69	.019	.076	.176	.300
70	.013	.060	.150	.268
71	.009	.047	.126	.237
72	.006	.036	.105	.209

Table A.5 (continued)

n = 7					n = 8			
x	m = 7	m = 8	m = 9	m = 10	x	m = 8	m = 9	m = 10
73	.003	.027	.087	.182	80	.117	.240	.381
74	.002	.020	.071	.157	81	.097	.212	.348
75	.001	.014	.057	.135	82	.080	.185	.317
76	.001	.010	.045	.115	83	.065	.161	.286
77	.000	.007	.036	.097	84	.052	.138	.257
78		.005	.027	.081	85	.041	.118	.230
79		.003	.021	.067	86	.032	.100	.204
80		.002	.016	.054	87	.025	.084	.180
81		.001	.011	.044	88	.019	.069	.158
82		.001	.008	.035	89	.014	.057	.137
83		.000	.006	.028	90	.010	.046	.118
84		.000	.004	.022	91	.007	.037	.102
85			.003	.017	92	.005	.030	.086
86			.002	.012	93	.003	.023	.073
87			.001	.009	94	.002	.018	.061
88			.001	.007	95	.001	.014	.051
89			.000	.005	96	.001	.010	.042
90			.000	.003	97	.001	.008	.034
91			.000	.002	98	.000	.006	.027
92				.002	99	.000	.004	.022
93				.001	100	.000	.003	.017
94				.001	101		.002	.013
95				.000	102		.001	.010
96				.000	103		.001	.008
97				.000	104		.000	.006
98				.000	105		.000	.004
					106		.000	.003
					107		.000	.002
					108		.000	.002
					109			.001
					110			.001
					111			.000
					112			.000
					113			.000
					114			.000
					115			.000
					116			.000

n = 8			
x	m = 8	m = 9	m = 10
68	.520		
69	.480		
70	.439		
71	.399		
72	.360	.519	
73	.323	.481	
74	.287	.444	
75	.253	.407	
76	.221	.371	.517
77	.191	.336	.483
78	.164	.303	.448
79	.139	.271	.414

Table A.5 (continued)

n = 9			n = 9			n = 10	
x	m = 9	m = 10	x	m = 9	m = 10	x	m = 10
86	.500		122	.000	.004	121	.124
87	.466		123	.000	.003	122	.109
88	.432		124	.000	.002	123	.095
89	.398		125	.000	.001	124	.083
90	.365	.516	126	.000	.001	125	.072
91	.333	.484	127		.001	126	.062
92	.302	.452	128		.000	127	.053
93	.273	.421	129		.000	128	.045
94	.245	.390	130		.000	129	.038
95	.218	.360	131		.000	130	.032
96	.193	.330	132		.000	131	.026
97	.170	.302	133		.000	132	.022
98	.149	.274	134		.000	133	.018
99	.129	.248	135		.000	134	.014
100	.111	.223				135	.012
101	.095	.200				136	.009
102	.081	.178				137	.007
103	.068	.158				138	.006
104	.057	.139				139	.004
105	.047	.121				140	.003
106	.039	.106				141	.003
107	.031	.091				142	.002
108	.025	.078				143	.001
109	.020	.067				144	.001
110	.016	.056				145	.001
111	.012	.047				146	.001
112	.009	.039				147	.000
113	.007	.033				148	.000
114	.005	.027				149	.000
115	.004	.022				150	.000
116	.003	.017				151	.000
117	.002	.014				152	.000
118	.001	.011				153	.000
119	.001	.009				154	.000
120	.001	.007				155	.000
121	.000	.005					

n = 10	
x	m = 10
105	.515
106	.485
107	.456
108	.427
109	.398
110	.370
111	.342
112	.315
113	.289
114	.264
115	.241
116	.218
117	.197
118	.176
119	.157
120	.140

Adapted from Table B of *A Nonparametric Introduction to Statistics*, by C. H. Kraft and C. van Eeden, Macmillan, New York, 1968, with the permission of the authors and the publisher. Copyright © 1968, by the Macmillan Company.

Table A.6. Upper tail probabilities for the null distribution of the Ansari-Bradley \mathcal{W} statistic: $2 \leq m \leq n, (m+n) \leq 20$

For given m and n , the table entry for the point x is $P_0\{\mathcal{W} > x\}$. Under these conditions, if x is such that $P_0\{\mathcal{W} > x\} = \alpha$, then $\omega_2(\alpha, m, n) = x$. On the other hand, if x is such that $P_0\{\mathcal{W} > x\} = 1 - \alpha$, then $P_0\{\mathcal{W} < (x-1)\} = P_0\{\mathcal{W} < x\} = [1 - P_0\{\mathcal{W} > x\}] = [1 - (1 - \alpha)] = \alpha$, and $\omega_1(\alpha, m, n) = (x-1)$.

m = 2									
x	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n = 10
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	.8333	.9000	.9333	.9524	.9643	.9722	.9778	.9818	.9848
4	.6667	.5000	.6667	.7619	.8214	.8611	.8889	.9091	.9242
5		.2000	.3333	.5238	.6429	.7222	.7778	.8182	.8485
6			.0667	.2381	.3571	.5000	.6000	.6727	.7273
7				.0952	.1786	.3056	.4000	.5091	.5909
8					.0357	.1389	.2222	.3273	.4091
9						.0556	.1111	.2000	.2727
10							.0222	.0909	.1515
11								.0364	.0758
12									.0152

m = 2								
x	n = 11	n = 12	n = 13	n = 14	n = 15	n = 16	n = 17	n = 18
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	.9872	.9890	.9905	.9917	.9926	.9935	.9942	.9947
4	.9359	.9451	.9524	.9583	.9632	.9673	.9708	.9737
5	.8718	.8901	.9048	.9167	.9265	.9346	.9415	.9474
6	.7692	.8022	.8286	.8500	.8676	.8824	.8947	.9053
7	.6538	.7033	.7429	.7750	.8015	.8235	.8421	.8579
8	.5000	.5714	.6286	.6750	.7132	.7451	.7719	.7947
9	.3590	.4286	.5048	.5667	.6176	.6601	.6959	.7263
10	.2308	.2967	.3714	.4333	.5000	.5556	.6023	.6421
11	.1410	.1978	.2667	.3250	.3897	.4444	.5029	.5526
12	.0641	.1099	.1714	.2250	.2868	.3399	.3977	.4474
13	.0256	.0549	.1048	.1500	.2059	.2549	.3099	.3579
14		.0110	.0476	.0833	.1324	.1765	.2281	.2737
15			.0190	.0417	.0809	.1176	.1637	.2053
16				.0083	.0368	.0654	.1053	.1421
17					.0147	.0327	.0643	.0947
18						.0065	.0292	.0526
19							.0117	.0263
20								.0053